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3D Eikonal Solver in Tilted TI Media

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SUMMARY

In this paper, I developed an eikonal equation 3D-solver for a tilted TI media. The method is based on an iterative approximation of the vertical slowness. The approximation technique is inserted into a second-order expanding box finite-difference upwind ENO algorithm. Numerical examples of qP-wave traveltime computation for tilted TI media with large anellipticity illustrate the accuracy and efficiency of the proposed eikonal solver.

Introduction

Anisotropic traveltime computation is a key step in seismic data processing, such as Kirchhoff depth migration and velocity analysis. The calculus of traveltimes for an anisotropic medium is a time extensive process because generally the eikonal is a sixth-order polynomial equation. With finite-difference or ray tracing methods for computing traveltimes, this equation should be solved numerically at each step (Eaton, 1993). Furthermore, for this type of media, computation of the time update direction in any upwind-scheme is much more diffcalt since it coincides not with the slowness vector, but with the normal to the slowness surface, i.e. with the directin of energy propagation (Dellinger and Symes, 1997). To simplify computing traveltimes in anisotropic media, various methods have been suggested. Most frequently elastic models are used for which the eikonal is a biquadratic polynomial equation. The simplest of them are VTI-media (Schoenberg and de Hoop, 2000; Qian and Symes, 2002) or acoustic VTI media (Alkhalifah, 2000). However, in actual case, we have to introduce weak anisotropy models or TI models with a tilted anisotropy axis. The slowness surfaces for such media satisfy a forth-order equation. In order not to solve these equations for time extrapolation, perturbation methods are used (Ettrich and Gajewski, 1998; Alkhalifah, 2002; Schneider, 2003; Soukina et al., 2003; Jiao, 2005). In this case, for time updating, it is necessary to solve a first-order differential equation. The coefficients of this equation depend on the traveltime derivatives of the reference medium. Therefore, they should be calculated with a sufficient accuracy. The eikonal equation is often considered a Hamilton-Jacobi equation and to obtain its viscosity upwind-ENO solution, the expanding wavefront or expanding box scheme is used (Dellinger and Symes, 1997; Qian et al., 2001; Kim, 1999; Wang, 2004). The aim of this article is to combine the above approaches and to obtain an accurate and effective method for traveltime extrapolation in titled TI-media.

Theory

The qP- slowness surface is insensitive to changing values of qS-wave velocities, even for relatively strong anisotropy (Alkhalifah, 1998; Schoenberg and de Hoop, 2000). Therefore, for TI-media, let us suppose that qS-wave velocity is equal to zero along the symmetry axis. Such media are called acoustic. This assumption does not give rise to significant deviations in qP-wave velocities, while the eikonal equation becomes much simpler (Alkhalifah, 1998):

$$(1+2\varepsilon)(p_1^2+p_2^2)+p_3^2-2(\varepsilon-\delta)\mathrm{V}_{po}^2(p_1^2+p_2^2)p_3^2=\frac{1}{\mathrm{V}_{po}^2}, \tag{1}$$

where p_1, p_2, p_3 are the components of a slowness vector $\mathbf{p}$; ε, δ are Thomsen's parameters; V_{p0} is the qP-wave velocity along the symmetry axis.

In order to derive the slowness surface equation for tilted acoustic TI media with a symmetry axis $\mathbf{u}_3=(\cos\varphi\sin\alpha,\, \sin\varphi\sin\alpha,\, \cos\alpha)^T$, let us introduce the column vectors $\mathbf{u}_1=(\cos\varphi\cos\alpha,\, \sin\varphi\cos\alpha,\, -\sin\alpha)^T$ and $\mathbf{u}_2=(-\sin\varphi,\, \cos\varphi,\, 0)^T$. These vectors form a basis for the 3D space. Let us pass to a new basis by the change of variables: $\mathbf{P\,w}=\mathbf{p}$, where $\mathbf{w}=(w_1, w_2, w_3)^T$, $\mathbf{p}=(p_1, p_2, p_3)^T$. The matrix $\mathbf{P}=(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$ consists of the column vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$.

As $\mathbf{P}$ is an orthogonal matrix, $\mathbf{P}^{-1}=\mathbf{P}^T$ and, therefore,

$$\begin{cases} w_1 = p_1\cos\varphi\cos\alpha + p_2\sin\varphi\cos\alpha - p_3\sin\alpha \\ w_2 = -p_1\sin\varphi + p_2\cos\varphi \\ w_3 = p_1\cos\varphi\sin\alpha + p_2\sin\varphi\sin\alpha + p_3\cos\alpha. \end{cases} \tag{2}$$

In a coordinate system (w_1, w_2, w_3) the symmetry axis of the TI-medium is vertical and coincides with the vector $\mathbf{e}_3=(0,\, 0,\, 1)^T$. Therefore, the slowness surface equation for the acoustic tilted TI medium can be described with the formula:

$$F(p_1,p_2,p_3)=A(p_1,p_2,p_3)-d\,B(p_1,p_2,p_3)=0\,, \tag{3}$$

where

$$d = \varepsilon - \delta\,, \qquad A(p_1,p_2,p_3) = (1+2\varepsilon)\left(w_1^2 + w_2^2\right) + w_3^2 - \frac{1}{V_{p_0}^2}\,, \qquad B(p_1,p_2,p_3) = 2V_{p0}^2\left(w_1^2 + w_2^2\right)w_3^2\,.$$

In the case of $\varepsilon = \delta$, the equation (3) can be simplified:

$$F_e(p_1,p_2,p_3) = (1+2\varepsilon)\left(w_1^2 + w_2^2\right) + w_3^2 - \frac{1}{V_{p_0}^2} = 0\,, \tag{4}$$

which describes a slowness surface of an elliptical tilted TI-medium. This equation can easily be solved for the parameter p_3:

$$p_3^{(0)} = \frac{1}{q_3}\left[q_1\varepsilon\sin 2\alpha + \sqrt{q_1^2\varepsilon^2\sin^2 2\alpha - q_3\left(q_1^2 q_2 + (1+2\varepsilon)\,w_2^2 - 1/V_{p0}^2\right)}\right], \tag{5}$$

where

$$q_1 = p_1\cos\varphi + p_2\sin\varphi; \quad q_2 = 1 + 2\varepsilon\cos^2\alpha; \quad q_3 = 1 + 2\varepsilon\sin^2\alpha.$$

With fixed of p_1 and p_2, equation (3) describes the relation between p_3 and the coefficient of anellipticity $d = \varepsilon - \delta$. By analogy with the perturbation technique, we determine the differential of this dependence:

$$F'_{p3}\cdot\Delta p_3 + F'_d\Delta d = 0\,,$$

and than use it for an iterative improvement of the vertical slowness p_3:

$$p_3^{(i+1)} = p_3^{(i)} + (d - d^{(i)})\frac{B(p_1,p_2,p_3^{(i)},d^{(i)})}{\partial F/\partial p_3\left(p_1,p_2,p_3^{(i)},d^{(i)}\right)} \quad \text{and} \quad d^{(i+1)} = \frac{A(p_1,p_2,p_3^{(i)},d^{(i)})}{B(p_1,p_2,p_3^{(i)},d^{(i)})}\,, \tag{6}$$

where

$$\frac{\partial F}{\partial p_3} = -2(1+2\varepsilon)w_1\sin\alpha + 2w_3\cos\alpha - 4dV_{p_0}^2 w_3\left[-w_1w_3\sin\alpha + \left(w_1^2 + w_2^2\right)\cos\alpha\right].$$

At the first step of the approximation process, we suppose that $d = 0$. From this it is follows that $p_3 = p_3^{(0)}$.

Calculating the function $p_3 = p_3(p_1,p_2)$ for tilted TI media is a key step of the suggested traveltime computation using the upwind ENO Runge-Kutta scheme with the expanding box strategy. The direction of energy propagation in plane $p_i = const$ and the maxima p_i^* (i = 1,2) of function $p_3 = p_3(p_1,p_2)$ are entirely defined by the derivatives $\partial F/\partial p_i$. For instance, if $\partial F/\partial p_1 < 0$, the energy propagates leftward, and if $\partial F/\partial p_1 > 0$, the energy propagates rightward. In the case of $\partial F/\partial p_1\left(p_1^*\right) = 0$, the energy propagates vertically downwards.

The of horizontal derivatives for the acoustic tilted TI media are calculated with the formulas:

$$\frac{\partial F}{\partial p_1} = 2(1+2\varepsilon)f_1 + 2w_3\cos\varphi\sin\alpha - 4(\varepsilon-\delta)V_{p0}^2 w_3\left(f_1w_3 + \left(w_1^2 + w_2^2\right)\cos\varphi\sin\alpha\right),$$

$$\frac{\partial F}{\partial p_2} = 2(1+2\varepsilon)f_2 + 2w_3\sin\varphi\sin\alpha - 4(\varepsilon-\delta)V_{p_0}^2 w_3\left(f_2w_3 + \left(w_1^2 + w_2^2\right)\sin\varphi\sin\alpha\right),$$

where

$$f_1 = w_1\cos\varphi\cos\alpha - w_2\sin\varphi\,,$$

and

$$f_2 = w_1\sin\varphi\cos\alpha + w_2\cos\varphi.$$

Analytical study of the convergence speed of this iterative algorithm is quite difficult. However, results of its application for different media demonstrate that weak anisotropic media and media with negative anellipticity $(\varepsilon - \delta < 0)$ require only one step of iterations, while media with strong positive anellipticity $(\varepsilon - \delta > 0)$ require two or three steps.

Let us illustrate the iterative scheme with two media from (Shoenberg and de Hoop, 2000). In this paper, VTI-media with strong anellipticity are presented. We consider TTI media with the same parameters except for the symmetry axis which is tilted in our case at the angle of 60^0 to the vertical. The squared velocity moduli in (km/s)^2 are $c_{11} = 14.47$, $c_{33} = 9.57$, and $c_{55} = 2.28$, while Thomsen's parameter is $\varepsilon = 0.256$.

Let us consider two types of anisotropic media:

a) $\delta = -0{,}324$, i.e. $c_{13} = 0{,}548$, $\varepsilon - \delta = 0{,}58$ is a TTI medium with strong positive anellipticity;

b) $\delta = 0{,}711$, i.e. $c_{13} = 10{,}05$, $\varepsilon - \delta = -0{,}455$ is a TTI medium with strong negative anellipticity.

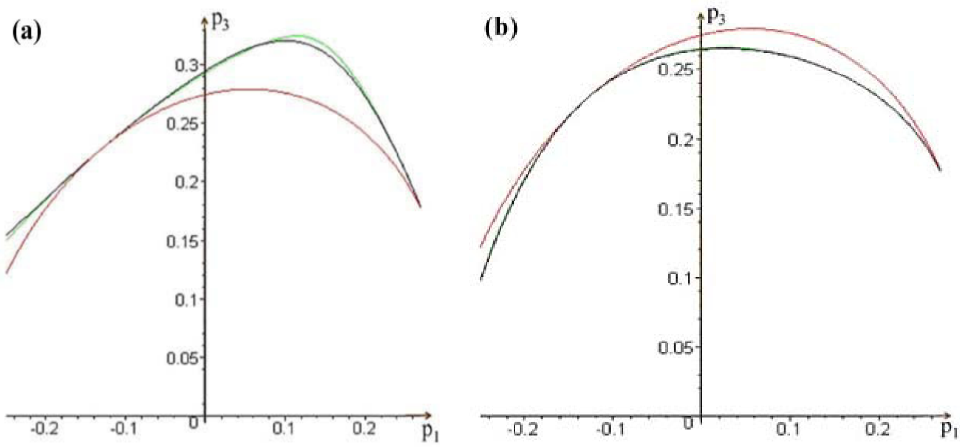

Figure 1. Exact slowness curve (green), elliptical approximation (red), first approximation (black): (a) $\varepsilon - \delta = 0.58$; (b) $\varepsilon - \delta = -0.455$.

In Fig.1, one can see the vertical slowness curves in the plane $p_2 = 0$ and their approximations for the above two models. The exact curves in this figure satisfy the equation

$$F(p_1, p_2 p_3) = \left[c_{11}(w_1^2 + w_2^2) + c_{33}w_3^2 - 1\right]\cdot\left[c_{55}(w_1^2 + w_2^2 + w_3^2) - 1\right] + 2(\varepsilon - \delta)c_{33}(c_{33} - c_{55})(w_1^2 + w_2^2)w_3^2 = 0\,.$$

After the second step of approximation, the vertical slowness p_3 practically coincides with its acoustic approximation calculated with equation (4). From this figure, it is also visible that the difference between the exact curve and its acoustic approximation is insignificant.

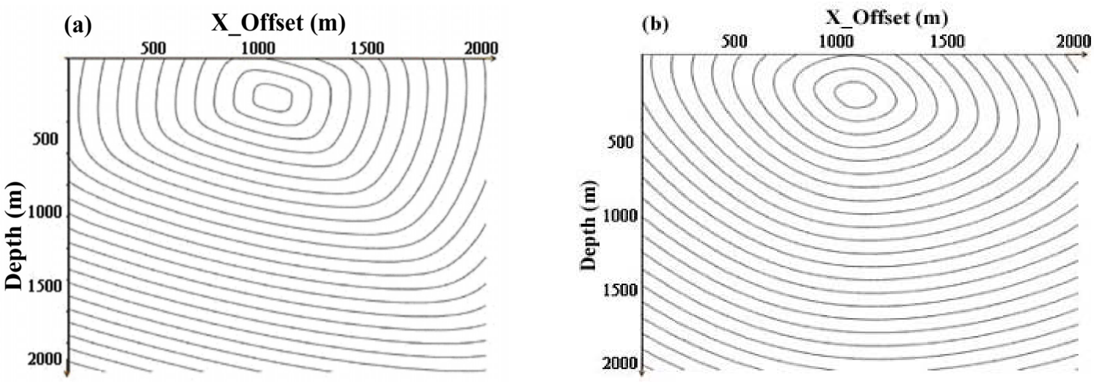

Figure 2. Time fields for TI media with the symmetry axis tilted at the angle of 60^0 to the vertical: (a) $\varepsilon - \delta = 0.58$; (b) $\varepsilon - \delta = -0.455$.

In Fig.2, results of application of the upwind ENO Runge-Kutta second order scheme to the above two models are demonstrated. In this scheme, the vertical slowness is calculated using the described method. When using an expanding box scheme, we update the traveltime at the box side of its minimum.

The speed of convergence of the approximation process for the tilted TI media with negative anellipticity is much higher than that of the media with positive anellipticity.

The result of testing the method with various model data demonstrates that the method is quite stable and time-efficient and may be included in migration, velocity analysis, etc.

Conclusions

An iteration scheme for calculating the vertical slowness in tilted TI media is developed. It utilizes a linearized eikonal equation from the perturbation theory. The scheme is rapidly converging and requires 1–3 steps of iteration for obtaining traveltimes with enough accuracy.

Unlike the conventional iterative algorithms from the perturbation theory, this scheme:

- does not require the calculation and storage of temporary traveltimes;
- does not require accurate estimates of the horizontal derivatives of the traveltime field;
- does not require determining the upwind update direction at each iteration step.

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