P091 PERTURBATION OF THE ELASTIC SCATTERING MATRIX IN ANISOTROPIC MEDIA

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Abstract

An approximate scattering matrix for an interface separating two arbitrary anisotropic half-spaces is obtained. The matrix depends on perturbation of elastic coefficients, densities, and directions of wave propagation. Compact linearized formulas for the reflection/transmission coefficients are derived for a weak-contrast interface and an anisotropic background. In case of an isotropic background the PP reflection/transmission coefficients are reduced to simple analytic equations expressed in terms of independent linear combinations of perturbations of normalized elastic coefficients and densities, which can be determined from observed data.

Introduction

Approximate formulas for the reflection/transmission (R/T) coefficients are widely used to invert seismic data into the physical properties of a media, especially in amplitude-versus-offset (AVO) analysis and amplitude-versus-azimuth (AVA) analysis, and to study the behavior of these coefficients in the neighborhood of a given direction.

Linearized formulas of the PP reflection/transmission coefficients for an arbitrarily weak-anisotropic perturbation of an isotropic medium were derived by Vavrycuk and Psencýk (1998), Psencýk and Vavrycuk (1998), Zillmer (1998). Klimes (2003) obtained approximate equations of the scattering matrix of an arbitrary oriented weak-contrast interface. Formulas of the PP reflection/transmission coefficients for a strong-contrast interface separating two weakly anisotropic half-spaces where derived by Zillmer (1997).

Theory

Following these studies, let us consider a medium with two anisotropic half-spaces separated by plane $x_3 = 0$ and denote the densities and the elastic parameters at the both half-spaces as $\rho^{(i)}$ and $\lambda^{(i)}_{mp,nq}$, i=1,2, respectively.

Plane elastic waves in the half-spaces i=1, 2 with a common frequency ω and an identical projection of a slowness vectors $\mathbf{s}^{(i)}_\alpha = \left(s_1; s_2; s^{(i)}_{3\alpha}\right)$ onto the plane $x_3 = 0$ are the sum of the waves with different modes α and can be expressed in the following way:

$$\mathbf{u}^{(i)}(\mathbf{r},t) = \sum_\alpha b^{(i)}_\alpha \mathbf{a}^{(i)}_\alpha \exp\{j\omega(\mathbf{s}\cdot\mathbf{r} - t)\}, \tag{1}$$

where $\mathbf{a}^{(\mathbf{i})}_\alpha$ is a unit polarization vector, $b^{(\mathbf{i})}_\alpha$ is a scalar amplitude. The subscript α denotes a type of waves: α=1, 2, 3 correspond to the down-going qP, S1, S2 waves, while α=4, 5, 6 correspond to up-going qP, S1, S2 waves, respectively.

Let $\mathbf{C}^{(i)}_{mn}$ denote a 3x3 matrix whose elements are the elastic parameters: $\mathbf{C}^{(i)}_{mn}[p,q] = \lambda^{(i)}_{mp,nq}$. An α-type wave presses per unit area of the interface $x_3 = 0$ with the force $\mathbf{f}^{(i)}_\alpha = -j\omega\,\mathbf{l}^{(i)}_\alpha$, where

$$\mathbf{l}^{(i)}_\alpha = -\left(s_1\mathbf{C}^{(i)}_{31} + s_2\mathbf{C}^{(i)}_{32} + s^{(i)}_{3\alpha}\mathbf{C}^{(i)}_{33}\right)\mathbf{a}^{(i)}_\alpha. \tag{2}$$

Let $\mathbf{v}^{(i)}_\alpha$ and $\mathbf{n}^{(i)}_\alpha$ denote 6-dimensional column and row vectors respectively, determined in the following way: $\mathbf{v}^{(i)}_\alpha = \begin{pmatrix}\mathbf{a}^{(i)}_\alpha \\ \mathbf{l}^{(i)}_\alpha\end{pmatrix}$, $\mathbf{n}^{(i)}_\alpha = \left(\mathbf{l}^{(i)T}_\alpha;\ \mathbf{a}^{(i)T}_\alpha\right)$, where T stands for transposition.

In each of the half-spaces i=1,2 let $\left\{\mathbf{a}^{(i)}_\alpha, \rho^{(i)}\right\}$ are the eigenpairs of symmetrical Christoffel matrixes $\mathbf{G}^{(\mathrm{i})}_\alpha = \sum_{p,q=1,2} s_p s_q \mathbf{C}^{(i)}_{pq} + \sum_{p=1,2} s_p s^{(i)}_{3\alpha}\left(\mathbf{C}^{(i)}_{p3} + \mathbf{C}^{(i)}_{3p}\right) + \left(s^{(i)}_{3\alpha}\right)^2 \mathbf{C}^{(i)}_{33}$, which depend on the wave type, α :

$$\mathbf{G}^{(\mathrm{i})}_\alpha \mathbf{a}^{(i)}_\alpha = \rho^{(i)}\ \mathbf{a}^{(i)}_\alpha. \tag{3}$$

Let $\mathbf{C}_{33}^{(i)}$, i=1,2, be nonsingular. This is usually the case since these matrixes possess relatively great elements $\lambda_{13,13}^{(i)}$, $\lambda_{23,23}^{(i)}$, $\lambda_{33,33}^{(i)}$ on their principal diagonals and relatively small ones elsewhere.

Equation (2) can be expressed as

$$\mathbf{C}_{\mathbf{33}}^{(i)-1}\left(s_1\mathbf{C}_{31}^{(i)} + s_2\mathbf{C}_{32}^{(i)}\right)\mathbf{a}_{\alpha}^{(i)} + \mathbf{C}_{\mathbf{33}}^{(i)-1}\mathbf{l}_{\alpha}^{(i)} = -s_{3\alpha}^{(i)}\mathbf{a}_{\alpha}^{(i)} . \tag{4}$$

Substituting (2) and (4) in (3) yields:

$$\sum_{p,q=1,2} s_p s_q\left(\mathbf{C}_{p3}^{(i)}\mathbf{C}_{\mathbf{33}}^{(i)-1}\mathbf{C}_{3q}^{(i)} - \mathbf{C}_{pq}^{(i)}\right)\mathbf{a}_{\alpha}^{(i)} + \rho^{(i)}\ \mathbf{a}_{\alpha}^{(i)} + \left(s_1\mathbf{C}_{13}^{(i)} + s_2\mathbf{C}_{23}^{(i)}\right)\mathbf{C}_{\mathbf{33}}^{(i)-1}\mathbf{l}_{\alpha}^{(i)} = -s_{3\alpha}^{(i)}\mathbf{l}_{\alpha}^{(i)} . \tag{5}$$

Jointly, equations (4) and (5) demonstrate that the vectors $\mathbf{v}_{\alpha}^{(i)}$ and $\mathbf{n}_{\alpha}^{(i)}$ are the right and left eigenvectors of 6x6 matrix

$$\mathbf{M}^{(i)} = -\begin{pmatrix} \mathbf{A}^{(i)} & \mathbf{C}_{33}^{(i)-1} \\ \mathbf{B}^{(i)} & \mathbf{A}^{(i)\mathbf{T}} \end{pmatrix} \tag{6}$$

associated with their eigenvalues, $s_{3\alpha}^{(i)}$, i.e.

$$\mathbf{M}^{(i)}\mathbf{v}_{\alpha}^{(i)} = s_{3\alpha}^{(i)}\mathbf{v}_{\alpha}^{(i)} \quad \text{and} \quad \mathbf{n}_{\alpha}^{(i)}\mathbf{M}^{(i)} = s_{3\alpha}^{(i)}\mathbf{n}_{\alpha}^{(i)} , \tag{7}$$

where $\mathbf{A}^{(i)} = \mathbf{C}_{\mathbf{33}}^{(i)-1}\left(s_1\mathbf{C}_{31}^{(i)} + s_2\mathbf{C}_{32}^{(i)}\right)$, $\mathbf{B}^{(i)} = \sum_{p,q=1,2} s_p s_q\left(\mathbf{C}_{p3}^{(i)}\mathbf{C}_{\mathbf{33}}^{(i)-1}\mathbf{C}_{3q}^{(i)} - \mathbf{C}_{pq}^{(i)}\right) + \rho^{(i)}\ \mathbf{I}$.

Unlike the Christoffel matrixes $\mathbf{G}_{\alpha}^{(i)}$, the matrixes $\mathbf{M}^{(i)}$ do not contain the component $s_{3\alpha}^{(i)}$ of the slowness vector $\mathbf{s}_{\alpha}^{(i)}$. For this reason, they are referred not to a concrete wave mode, but to wave packets as a whole.

Let $\mathbf{E}^{(i)}$ be a matrix consisting of the column eigenvectors $\mathbf{v}_{\alpha}^{(i)}$ of the matrix $\mathbf{M}^{(i)}$. From (7) it follows that the inverse matrix $\left(\mathbf{E}^{(i)}\right)^{-1}$ consists of the following row eigenvectors:

$$\mathbf{w}_{\alpha}^{(i)} = \frac{\mathbf{n}_{\alpha}^{(i)}}{\mathbf{n}_{\alpha}^{(i)} \cdot \mathbf{v}_{\alpha}^{(i)}} = \frac{\mathbf{n}_{\alpha}^{(i)}}{2\mathbf{l}_{\alpha}^{(i)T} \cdot \mathbf{a}_{\alpha}^{(i)}} .$$

Let $\mathbf{X}=\left(\mathbf{v}_1^{(1)}\ \ \mathbf{v}_2^{(1)}\ \ \mathbf{v}_3^{(1)}\ \ \mathbf{v}_4^{(2)}\ \ \mathbf{v}_5^{(2)}\ \ \mathbf{v}_6^{(3)}\right)$ and $\mathbf{Y}=\left(\mathbf{v}_1^{(2)}\ \ \mathbf{v}_2^{(2)}\ \ \mathbf{v}_3^{(2)}\ \ \mathbf{v}_4^{(1)}\ \ \mathbf{v}_5^{(1)}\ \ \mathbf{v}_6^{(1)}\right)$ be the matrixes consisting of the column eigenvectors of matrixes $\mathbf{M}^{(i)}$, i=1,2, for all the types of plane waves coming to and leaving the interface, respectively, and **S** be the elastic scattering matrix containing all the R/T coefficients.

If **Y** is nonsingular, its columns form a basis in a 6-dimensional vector space. In this case the scattering matrix **S** can be found as $\mathbf{S} = \mathbf{Y}^{-1}\mathbf{X}$ (Aki and Richards, 1980). The calculation of **S** is thus reduced to calculation of the eigenvectors of the matrixes $\mathbf{M}^{(i)}$.

If the matrix $\mathbf{M}^{(\mathbf{i})}$ undergoes a perturbation, $\Delta\mathbf{M}^{(\mathbf{i})}$, caused by deviations of the elastic coefficients $\lambda_{mp,nq}^{(i)} \in \mathbf{C}_{pq}^{(i)}$, the components s_1 and s_2 of slowness vectors $\mathbf{s}_{\alpha}^{(i)}$, and densities $\rho^{(i)}$, the scattering matrix **S** obtains the following perturbation

$$\Delta\mathbf{S} = \mathbf{Y}^{-1} \cdot \Delta\mathbf{X} - \mathbf{Y}^{-1} \cdot \Delta\mathbf{Y} \cdot \mathbf{S} . \tag{8}$$

The matrixes $\mathbf{X}$ and $\mathbf{Y}$ can be expressed as follows:

$$\mathbf{X} = \begin{pmatrix} \mathbf{E}_{11}^{(1)} & \mathbf{E}_{12}^{(2)} \\ \mathbf{E}_{21}^{(1)} & \mathbf{E}_{22}^{(2)} \end{pmatrix}, \quad \mathbf{Y} = \begin{pmatrix} \mathbf{E}_{\mathbf{11}}^{(2)} & \mathbf{E}_{12}^{(1)} \\ \mathbf{E}_{21}^{(2)} & \mathbf{E}_{22}^{(1)} \end{pmatrix}, \quad \text{where} \quad \mathbf{E}^{(i)} = \begin{pmatrix} \mathbf{E}_{\mathbf{11}}^{(i)} & \mathbf{E}_{12}^{(i)} \\ \mathbf{E}_{\mathbf{21}}^{(i)} & \mathbf{E}_{22}^{(i)} \end{pmatrix} \tag{9}$$

is splitting the matrix $\mathbf{E}^{(i)}$ into the four 3x3-submatrixes $\mathbf{E}_{pq}^{(i)}$ (i, p, q=1,2).

According to the perturbation theory, if the matrix $\mathbf{M}^{(i)}$ has eigenvalues $s_{3p}^{(i)}$ of index 1, the first-order perturbation of the eigenvalues $\delta s_{3p}^{(i)}$ and eigenvectors $\delta\mathbf{v}_{p}^{(i)}$ of this matrix can be calculated as:

$$\delta s_{3p}^{(i)} = \mathbf{w}_{p}^{(i)}\Delta\mathbf{M}^{(i)}\mathbf{v}_{p}^{(i)} , \tag{10}$$

$$\delta\mathbf{v}_{p}^{(i)} = d_{pp}^{(i)}\mathbf{v}_{p}^{(i)} + \sum_{q\neq p} d_{qp}^{(i)}\mathbf{v}_{q}^{(i)} , \tag{11}$$

where

$$d_{qp}^{(i)} = \frac{\mathbf{w}_{q}^{(i)}\Delta\mathbf{M}^{(i)}\mathbf{v}_{p}^{(i)}}{s_{3p}^{(i)} - s_{3q}^{(i)}}, \quad \text{when} \quad p \neq q \quad \text{and} \quad s_{3q}^{(i)} \neq s_{3p}^{(i)} , \tag{12}$$

$$d_{pp}^{(i)} = 0, \quad \text{when} \quad p \neq q \quad \text{and} \quad s_{3q}^{(i)} = s_{3p}^{(i)},$$

$$d_{pp}^{(i)} = -\sum_{p\neq q} d_{qp}^{(i)}\left(\mathbf{a}_{\mathbf{q}}^{(i)T} \cdot \mathbf{a}_{\mathbf{p}}^{(i)}\right). \tag{13}$$

Note, equation (13) follows from the condition of normalization: $\left|\mathbf{a}_{p}^{(i)} + \delta\,\mathbf{a}_{p}^{(i)}\right| = 1$.

If the matrix $\mathbf{M}^{(i)}$ has a multiple eigenvalue, $s_{3p}^{(i)}$, the associated eigenvectors, $\mathbf{v}_{p_1}^{(i)},...\mathbf{v}_{p_k}^{(i)}$, should be selected to satisfy the equations $\mathbf{w}_{p_m}^{(i)}\Delta\mathbf{M}\mathbf{v}_{p_n}^{(i)} = 0$, with $m \neq n$.

Let $\widetilde{\Delta}\mathbf{G}_{qp}^{(i)}$ denote a deviation of the matrix

$$\mathbf{G}_{qp}^{(i)} = \sum_{m=1,2}\left(s_{3p}^{(i)}s_m\mathbf{C}_{m3}^{(i)} + s_{3q}^{(i)}s_m\mathbf{C}_{3m}^{(i)}\right) + \sum_{m,n=1,2}\mathbf{C}_{mn}^{(i)}s_m s_n + \mathbf{C}_{33}^{(i)}s_{3p}^{(i)}s_{3q}^{(i)} - \rho\,\mathbf{I}, \tag{14}$$

that emerges due to some changes in the elastic coefficients, the components s_1 and s_2 of the slowness vector, and the densities $\rho^{(i)}$, *with the values* $s_{3p}^{(i)}$ *and* $s_{3q}^{(i)}$ *being invariable*. Substituting $\mathbf{l}_q^{(i)} = -\mathbf{C}_{33}^{(i)}(\mathbf{A}^{(i)} + s_{3q}^{(i)})\mathbf{a}_q^{(i)}$ and $\mathbf{B}^{(i)} = \mathbf{A}^{(i)T}\mathbf{C}_{33}^{(i)}\mathbf{A}^{(i)} - \sum_{m,n=1,2}\mathbf{C}_{mn}^{(i)}s_m s_n + \rho\,\mathbf{I}$ in the expression $\mathbf{n}_q^{(i)}\Delta\mathbf{M}\mathbf{v}_p^{(i)}$ yields

$$\mathbf{n}_q^{(i)}\Delta\mathbf{M}\mathbf{v}_p^{(i)} = \mathbf{a}_q^{(i)T}\widetilde{\Delta}\mathbf{G}_{qp}^{(i)}\mathbf{a}_p^{(i)}. \tag{15}$$

From the formula $\mathbf{a}_q^{(i)T}\mathbf{G}_{qp}^{(i)}\mathbf{a}_q^{(i)} = \mathbf{a}_q^{(i)T}\mathbf{G}_{qq}^{(i)}\mathbf{a}_q^{(i)} + (s_{3p}^{(i)} - s_{3q}^{(i)})\mathbf{l}_q^{(i)T}\mathbf{a}_q^{(i)} = 0.5(s_{3p}^{(i)} - s_{3q}^{(i)})\mathbf{n}_q^{(i)}\mathbf{v}_q^{(i)}$ with $p \neq q$ and $s_{3q}^{(i)} \neq s_{3p}^{(i)}$ one can obtain

$$d_{qp}^{(i)} = \frac{\mathbf{a}_q^{(i)T}\widetilde{\Delta}\mathbf{G}_{qp}^{(i)}\mathbf{a}_p^{(i)}}{2\mathbf{a}_q^{(i)T}\mathbf{G}_{qp}^{(i)}\mathbf{a}_q^{(i)}}. \tag{15}$$

The similar equation, in the tensor notation, was recently obtained by Klimes (2003).

Let us introduce a matrix $\mathbf{D}^{(i)} = \left(d_{qp}^{(i)}\right)$ and split it into four 3x3-submatrixes $\mathbf{D}_{qp}^{(i)}$, p,q=1,2. Then equation (11) can be rewritten in a compact form:

$$\Delta\mathbf{E}^{(i)} = \mathbf{E}^{(i)}\cdot\mathbf{D}^{(i)}. \tag{16}$$

Using (8), (9), and (16), the following equations for the perturbations of the matrixes **X**, **Y** and **S** can be obtained:

$$\Delta\mathbf{X} = \mathbf{X}\begin{pmatrix}\mathbf{D}_{11}^{(1)} & \mathbf{0}\\ \mathbf{0} & \mathbf{D}_{22}^{(2)}\end{pmatrix} + \mathbf{Y}\begin{pmatrix}\mathbf{0} & \mathbf{D}_{12}^{(2)}\\ \mathbf{D}_{21}^{(1)} & \mathbf{0}\end{pmatrix}; \qquad \Delta\mathbf{Y} = \mathbf{X}\begin{pmatrix}\mathbf{0} & \mathbf{D}_{12}^{(1)}\\ \mathbf{D}_{21}^{(2)} & \mathbf{0}\end{pmatrix} + \mathbf{Y}\begin{pmatrix}\mathbf{D}_{11}^{(2)} & \mathbf{0}\\ \mathbf{0} & \mathbf{D}_{22}^{(1)}\end{pmatrix};$$

$$\Delta\mathbf{S} = \begin{pmatrix}\mathbf{0} & \mathbf{D}_{12}^{(2)}\\ \mathbf{D}_{21}^{(1)} & \mathbf{0}\end{pmatrix} + \mathbf{S}\begin{pmatrix}\mathbf{D}_{11}^{(1)} & \mathbf{0}\\ \mathbf{0} & \mathbf{D}_{22}^{(2)}\end{pmatrix} - \begin{pmatrix}\mathbf{D}_{11}^{(2)} & \mathbf{0}\\ \mathbf{0} & \mathbf{D}_{22}^{(1)}\end{pmatrix}\mathbf{S} - \mathbf{S}\begin{pmatrix}\mathbf{0} & \mathbf{D}_{12}^{(1)}\\ \mathbf{D}_{21}^{(2)} & \mathbf{0}\end{pmatrix}\mathbf{S}. \tag{17}$$

In a particular case of half-spaces with similar physical properties, the medium can be considered as perturbation of the homogeneous background. Since **S** for homogeneous media is an identity matrix, expression (17) for a weak-contrast interface becomes much simpler:

$$\Delta\mathbf{S} = \begin{pmatrix}\mathbf{D}_{11}^{(1)} - \mathbf{D}_{11}^{(2)} & -\mathbf{D}_{12}^{(1)} + \mathbf{D}_{12}^{(2)}\\ -\mathbf{D}_{21}^{(1)} + \mathbf{D}_{21}^{(2)} & \mathbf{D}_{22}^{(1)} - \mathbf{D}_{22}^{(2)}\end{pmatrix}. \tag{18}$$

Equation (18) demonstrates how a perturbation of the scattering matrix for a weak-contrast interface depends on differences in the elasticity and density of the half-spaces and a perturbation of the slowness.

The dependence of the PP reflection/transmission coefficients and qP-wave phase velocities squared on a perturbation of a homogeneous isotropic background can be written as:

$$k_R(n_1,n_2) = \frac{f_4}{4n_3^2} - \frac{f_2}{2} + \frac{n_3^2}{4}z_{33}; \tag{19}$$

$$k_T(n_1,n_2) = 1 + \frac{1}{4}\left(2 + \frac{1}{n_3^2}\right)f_4 + \frac{2n_3^2 - 1}{2}f_2 + \frac{n_3^2\left(2n_3^2 - 3\right)}{4}z_{33} + 2n_3 f_3; \tag{20}$$

$$v_{qP}^2(n_1,n_2) = \alpha^2\left(1 + f_4 + 2n_3^2 f_2 + n_3^4 a_{33} + 4n_3(f_1 + f_3)\right), \tag{21}$$

where $a_{ij} = \Delta c_{ij}/(\rho\alpha^2)$, $n_3(n_1,n_2) = \sqrt{1 - n_1^2 - n_2^2}$, $f_1 = a_{35}n_1 + a_{34}n_2$, $f_2 = z_{55}n_1^2 + 2z_{45}n_1n_2 + z_{44}n_2^2$, $f_3 = z_{13}n_1^3 + z_{56}n_1^2n_2 + z_{46}n_1n_2^2 + z_{23}n_2^3$, $f_4 = a_{11}n_1^4 + 4a_{16}n_1^3n_2 + 2z_{66}n_1^2n_2^2 + 4a_{26}n_1n_2^3 + a_{22}n_2^4$, $z_{33} = a_{33} + r_0$, $z_{44} = 2a_{44} - \varepsilon\,a_{23} + r$, $z_{55} = 2a_{55} - \varepsilon\,a_{13} + r$, $z_{66} = 2a_{66} + a_{12}$, $z_{45} = 2a_{45} - \varepsilon\,a_{36}$, $z_{23} = a_{24} - a_{34}$, $z_{13} = a_{15} - a_{35}$, $z_{56} = 2a_{56} - a_{34} + a_{14}$, $z_{46} = 2a_{46} - a_{35} + a_{25}$; $r = \left(\frac{4\beta^2}{\alpha^2} - 1\right)\frac{\Delta\rho}{\rho}$, $r_0 = \frac{2\Delta\rho}{\rho}$ and $\varepsilon = 1$ for reflected waves, $r = \frac{\Delta\rho}{\rho}$, $r_0 = \frac{2\Delta\rho}{\rho}$ and $\varepsilon = -1$ for transmitted waves, $r = 0$, $r_0 = 0$ and $\varepsilon = -1$ for phase velocities;

α and β are P- and S-wave velocities, ρ - is the density of a referent isotropic medium, Δc_{ij} are deviations of the elastic coefficients, $\mathbf{n} = (n_1, n_2, n_3)$ is a unit direction vector.

In expressions (19) - (21), the factors at a_{ij} and z_{ij} are linearly independent functions. For this reason, the parameters a_{ij} and z_{ij} from the above equations can be determined uniquely using a sufficiently large number of reflection/transmission coefficients and phase velocities. Analysis this equations allows us to draw a conclusion that only the elastic parameters and their linear combinations from Table 1 can be determined independently and uniquely.

Table 1: Elastic parameters recoverable from observed variables $k_R(n_1, n_2)$, $k_T(n_1, n_2)$, $v_{qP}^2(n_1, n_2)$

k_R, k_T, v_{qP}^2	a_{11}	a_{22}	$a_{33} + r_0$	a_{16}	a_{26}
k_R	$2a_{44} - a_{23} + r$	$2a_{55} - a_{13} + r$	$2a_{45} - a_{36}$	$2a_{66} + a_{12}$	
k_T, v_{qP}^2	$2a_{44} + a_{23} + r$ $2a_{46} - a_{35} + a_{25}$	$2a_{55} + a_{13} + r$ $2a_{56} - a_{34} + a_{14}$	$2a_{45} + a_{36}$ $a_{24} - a_{34}$	$2a_{66} + a_{12}$ $a_{15} - a_{35}$	
v_{qP}^2	a_{34}	a_{35}			

Equations (19)-(21) for various directions $\mathbf{n}_i$ represent an overdetermined system of linear equations in the unknowns from Table 1. Let $\mathbf{Fx} = \mathbf{b}$ be a matrix representation of the above system of equations.

In Figure 1, the eigenvalues of the matrix $\left(\mathbf{F}^T\mathbf{F}\right)^{1/2}$ are displayed for various combinations of equations (19)-(21). The eigenvalues were normalized and arranged in a non-ascending order. The directions $\mathbf{n}_i$ uniformly covered an incidence interval $0 < \theta < \pi/4$ and an azimuth interval $0 < \varphi < 2\pi$. For simplicity, it was supposed that $\Delta\rho = 0$.

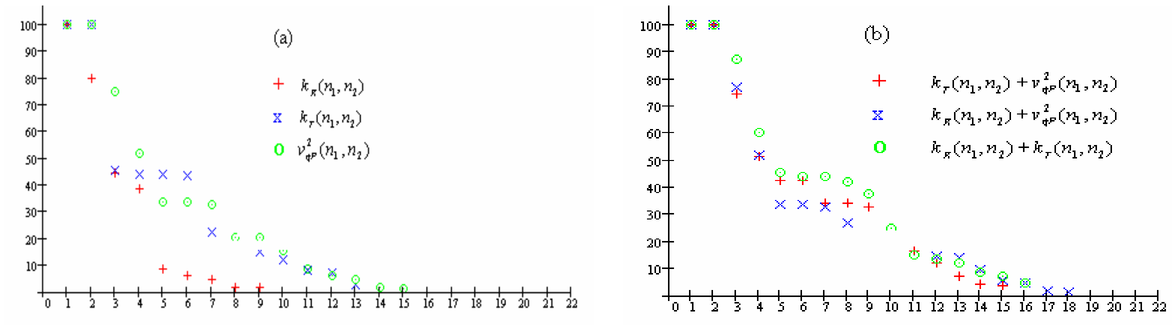

Figure 1: Normalized eigenvalues of the matrix $\left(\mathbf{F}^T\mathbf{F}\right)^{1/2}$.

Conclusions

Analysis of Table1 and Figure 1 allows us to draw a conclusion that 9, 13 and 15 elastic parameters or their linear combinations can be determined with the use of PP reflection coefficients (R), PP transmission coefficients (T) and qP-wave velocities (V) , respectively. Joint utilization of the observed data increases the number of recoverable parameters: (R+T) – 16; (R+V) – 18; (T+V) – 15 (the same parameters, that are determined using qP-wave velocities). In case of R+V it is impossible to restore deviations of the elastic coefficients Δc_{45} and Δc_{36} and to separate the linear combination $\Delta c_{12} + 2\Delta c_{66}$.

References

Aki, K., and Richards, P., 1980, Quantitative seismology: W. H. Freeman and Co.

Klimes L., 2003, Weak-contrast reflection–transmission coefficients in a generally anisotropic background: Geophysics, **68**, 2063–2072.

Psencýk, I., and Vavrycuk, V., 1998, Weak contrast *PP* wave displacement *R*/*T* coefficients in weakly anisotropic elastic media: Pageoph, **151**, 699–718.

Vavrycuk, V., Psencýk, I., 1998, PP-wave reflection coefficients in weakly anisotropic elastic media: Geophysics, **63**, 2129–2141.

Zillmer, M., Gajewski, D., and Kashtan, B. M., 1997, Reflection coefficients for weak anisotropic media: Geophys. J. Internat., **129**, 389–398.

——1998, Anisotropic reflection coefficients for a weak-contrast interface: Geophys. J. Internat., **132**, 159–166.