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An Initialization Method for Traveltime Calculation in TTI Media

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SUMMARY

In this paper, a fast method for computing both the group velocity and related slowness vector for a given direction in an acoustic TTI medium is developed. This method is based on an iterative Newton/Raphson-like procedure and can be used to initialize traveltime calculation and to determine the upwind stencil direction. Numerical examples of qP-wave group velocity computation in TTI media with large anellipticity illustrate the accuracy and efficiency of the proposed method.

Introduction

The initialization of a finite-difference scheme for traveltime calculation in an anisotropic medium contains the following three complications. First, wavefronts have a large curvature in the neighbourhood of the source (Qian et al. 2002). Therefore the spatial derivatives of traveltimes change rapidly near the source. So to solve this problem adaptive grid (Qian et al. 2002), locally uniform mesh refinement (Kim and Cook 1999) or spherical coordinates (Dellinger et al. 1997) methods are used. Second, the vectors of phase and group velocities may have different directions in an anisotropic medium. These facts should be taken into account when computing upwind direction and stencil orientation in any finite-difference scheme (Wang et al 2006). Third, it is difficult, in general, to describe group velocity analytically as a function of its direction of propagation. Therefore, this velocity is usually determined as a function of the vector of phase velocity (Faria et al. 1994) or as a function of the Hamiltonian derivatives with respect to the components of the slowness-vector (Qian et al. 2003). For weak anisotropic media, the group velocity as a function of its direction can be approximated by a few terms of the cosine Fourier transform. These formulae are often applied for TI-media (Faria et al. 1994, Kumar et al. 2004). For strong anisotropic media, Červený (2001) applied a method of anisotropic ray tracing to initialize traveltime calculations. Qian and Symes (2002) proposed a nonlinear iterative method to compute the group velocity and to initialize the traveltimes at grid points around the source. To simplify and speed up finite-difference traveltime computation in anisotropic media, acoustic TTI models with qS-wave velocities equal to zero along the symmetry axis are often applied (e.g. Alkhalifah, 1998; Zhang et al. 2002).

In this paper, a fast iterative method is developed for computing both the group velocity and the related slowness vector for a given direction in an acoustic TTI medium. It can be used to initialize the traveltime calculation in the neighbourhood of a source. It can also be used for defining extremes on a slowness surface. The coordinates of these extremes are then used to correctly determine the upwind stencil direction (Dellinger et al. 1997).

Theory

The equation of a qP slowness surface for an acoustic VTI medium can be obtained from the general eikonal equation for an arbitrary VTI medium by supposing the qS-wave velocity to equal zero along the symmetry axis:

$$a\left(p_1^2 + p_2^2\right) + p_3^2 - 2dV_{p_0}^2\left(p_1^2 + p_2^2\right)p_3^2 = \frac{1}{V_{p_0}^2} \qquad (1)$$

where $a = 1 + 2\varepsilon$, $d = \varepsilon - \delta$ and p_1, p_2, p_3 are the components of a slowness vector $\mathbf{p}$; ε, δ are Thomsen's parameters; V_{p_0} is the qP-wave velocity along the symmetry axis. In order to derive the slowness surface equation for a tilted acoustic TI medium with the symmetry axis $\mathbf{u}_3$, let us pass to a new basis by the change of variables: $\mathbf{Pw} = \mathbf{p}$, where $\mathbf{w} = (w_1, w_2, w_3)^T$, $\mathbf{p} = (p_1, p_2, p_3)^T$ and the superscripted T denotes transposition. The matrix $\mathbf{P} = (\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$ consists of the column vectors $\mathbf{u}_1 = (\cos\varphi\cos\alpha; \sin\varphi\cos\alpha; -\sin\alpha)^T$, $\mathbf{u}_2 = (-\sin\varphi; \cos\varphi; 0)^T$ and $\mathbf{u}_3 = (\cos\varphi\sin\alpha; \sin\varphi\sin\alpha; \cos\alpha)^T$.

Since $\mathbf{P}$ is orthogonal, $\mathbf{w} = \mathbf{P}^T\mathbf{p}$, which is equivalent to the system of equations:

$$\begin{cases} w_1 = p_1 \cos\varphi \cos\alpha + p_2 \sin\varphi \cos\alpha - p_3 \sin\alpha \\ w_2 = -p_1 \sin\varphi + p_2 \cos\varphi \\ w_3 = p_1 \cos\varphi \sin\alpha + p_2 \sin\varphi \sin\alpha + p_3 \cos\alpha . \end{cases} \quad (2)$$

The slowness surface of the acoustic tilted TI medium can therefore be described with the formula:

$$F = 0, \quad (3)$$

where $$F = a\left(w_1^2 + w_2^2\right) + w_3^2 - 2dV_{p_0}^2\left(w_1^2 + w_2^2\right)w_3^2 - \frac{1}{V_{p_0}^2}.$$

Let the direction of group velocity be described by the vector $\mathbf{n}_g = \left(n_1, n_2, n_3\right)^T$. In the coordinate system $\mathbf{w} = \left(w_1, w_2, w_3\right)^T$, this vector has coordinates $\mathbf{m}_g = \mathbf{P}^T \mathbf{n}_g$, and the symmetry axis of the TI medium is vertical.

In order to reduce the problem to a two-dimensional one, turn the vector $\mathbf{m}_g = \left(m_{1g}, m_{2g}, m_{3g}\right)$ around of the axis w_3 so that its coordinates become equal to $\mathbf{l}_g = \left(\sqrt{m_{1g}^2 + m_{2g}^2}; 0; m_{3g}\right)$. In case of $m_{g3} < 0$ it is also necessary to find the symmetrical analogue of this vector with respect to the plane $w_3 = 0$.

Let us find the coordinates of the related slowness vector $\mathbf{l}_s = \left(w_1; 0; w_3\right)$. Its coordinates satisfy simultaneously equation (3) with $w_2 = 0$ and the condition of collinearity for the vectors $\mathbf{l}_g$ and $\mathbf{f}_g = \left(F'_{w_1}; 0; F'_{w_3}\right)$. The last condition can formally be written as $G = 0$, where

$$G = F'_{w_1}\, m_{3g} - F'_{w_3}\, \sqrt{m_{1g}^2 + m_{1g}^2}\,. \quad (4)$$

The values of w_1 and w_2 are found by solving the system of equations (3) and (4) with a Newton/Raphson-like iterative procedure.

To obtain an initial approach to the slowness vector $\mathbf{l}_s$ we substitute in equation (3) $w_1 = w\cos\beta_i$, $w_2 = 0$ and $w_3 = w\sin\beta_i$ with different angles β_i, distributed between 0 and 90^0 in 5^0 increments. Then equation (3) is solved for all the above angles to obtain w. After that the slowness vectors $\mathbf{w}_i = \left(w_1;\, 0;\, w_3\right)$ and the vectors $\mathbf{f}_i = \left(F'_{w_1};\, 0;\, F'_{w_3}\right)$ are calculated. The vector $\mathbf{f}_i$ is collinear to the related vector of group velocity. Afterwards, among $\mathbf{f}_i$, vector $\mathbf{f}_j$ is found that has the minimal angular deviation from $\mathbf{l}_g$. In the first step of the approximation process, the slowness vector $\mathbf{l}_s$ is supposed to satisfy the equation $\mathbf{l}_{s,0} = \mathbf{w}_j$.

To compute the slowness vector $\mathbf{l}_s$ related to the group velocity vector $\mathbf{l}_g$, the following iterative process is utilized:

$$\mathbf{l}_{s,n+1} = \mathbf{l}_{s,n} - \mathbf{A}^{-1}\mathbf{v}_n,$$

where $\mathbf{v}_n = \left(F, G\right)^T$. The functions F and G are calculated with $w_1 = w_{1,n}$, $w_2 = 0$, and $w_3 = w_{3,n}$. The matrix $\mathbf{A}$ consists of elements

$$a_{11} = \frac{\partial F}{\partial w_1}\ ,\ a_{12} = \frac{\partial F}{\partial w_3}\ ,\ a_{21} = \frac{\partial^2 F}{\partial w_1^2} w_3 - \frac{\partial^2 F}{\partial w_1 \partial w_3} w_1\ ,\ a_{22} = \frac{\partial^2 F}{\partial w_1 \partial w_1} w_3 - \frac{\partial^2 F}{\partial w_3^2} w_1 .$$

With the slowness vector $\mathbf{l}_s$ obtained, the group velocity along the direction $\mathbf{l}_g$ can be estimated using equality $v_g = (\mathbf{l}_g \cdot \mathbf{l}_s)^{-1}$.

Let us illustrate the iterative initializing scheme on two acoustic VTI media with strong positive and negative anellipticities.

In the first example (Figure 1a), conceder a VTI medium with $V_{p_0} = 3000\, m/s$, $\varepsilon = 0.3$, $\delta = -0.45$, and find the angle of the slowness vector $\mathbf{l}_s$, the values of the group and phase velocities at the angle of energy propagation $\varphi_g = 71^0$.

As result of each step of calculation, approximate values of the slowness vector $\mathbf{l}_{si}$ and group velocity v_g are obtained. The relative errors a_i of the first three steps of approximation of group velocity are $a_1 = 4.58\%$, $a_2 = 0.21\%$, and $a_3 = 0.001\%$. After the third step of calculation, the angle of slowness vector is $\varphi_f = 41{,}8^0$. This angle differs considerably from the angle of energy propagation $\varphi_g = 71^0$. The estimated values of the group and phase velocities are: $v_g = 3090\, m/s$ and $v_f = 2698\, m/s$, respectively.

In the second example (Figure 1b), conceder a VTI medium with $V_{p_0} = 3000\, m/s$, $\varepsilon = -0.3$, $\delta = 0.45$, and the angle of energy propagation $\varphi_g = 43^0$ and perform the same iterative operations. In this case the first three relative errors of group velocity estimates are $a_1 = 5.62\%$, $a_2 = 0.58\%$, and $a_3 = 0.007\%$. The angle of the slowness vector is $\varphi_f = 83.8^0$. The estimated values of group and phase velocities are $v_g = 2646\, m/s$ and $v_f = 2004\, m/s$, respectively.

In Figure 2a and Figure 2b, the traveltimes in the neighbourhood of the source for both examples are displayed. These traveltimes were calculated using the group velocities obtained by the application of the above algorithm.

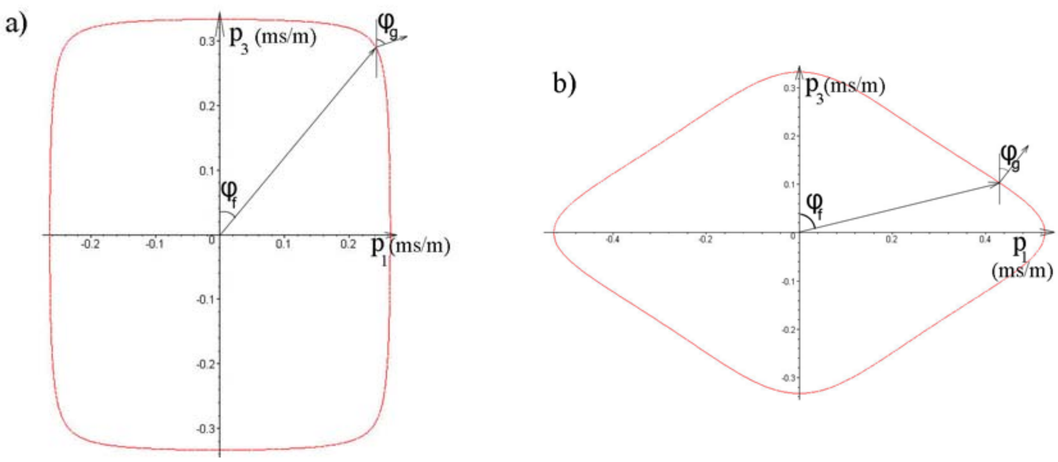

Figure 1. Slowness curves for acoustic VTI media with qP velocity along the vertical axis $V_{p_0} = 3000\, m/s$ and large positive and negative value of the anellipticity parameters.
(a) $\varepsilon = 0.3$ and $\delta = -0.45$; (b) $\varepsilon = -0.3$ and $\delta = 0.45$.

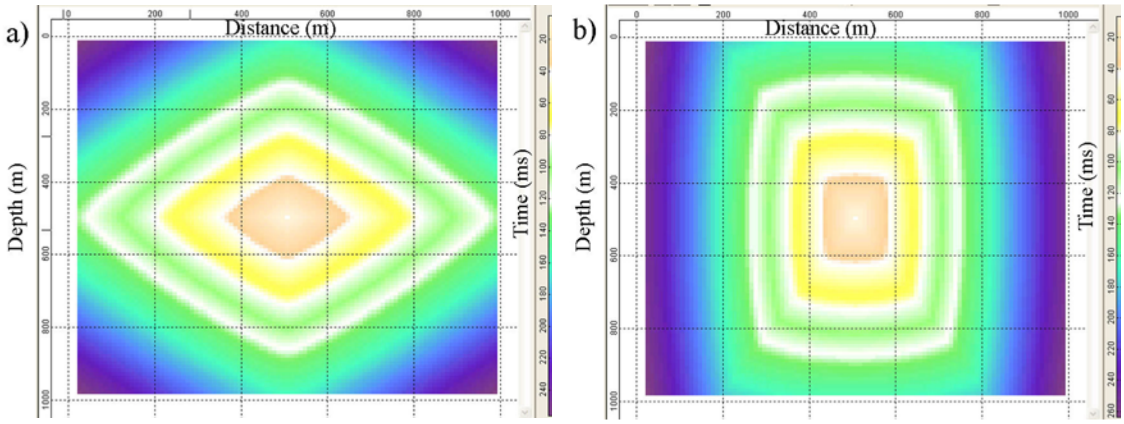

Figure 2. Traveltimes for homogeneous VTI media with large (a) positive and (b) negative anellipticity parameters. qP velocity and Thomsen's parameters are the same as in Figure 1.

Conclusions

A fast iterative algorithm for calculating the group velocity and related slowness vector for a given energy propagation direction in an anisitropic TTI medium is obtained. This algorithm can be used to initialize any finite-difference scheme for traveltime computation. Also, it can be used to correctly determine upwind stencil direction in upwind finite-difference scheme applied to TTI medium.

The algorithm is robust, converges rapidly and requires only 2-3 steps to obtain group velocity and traveltime estimates with sufficient accuracy.

References

Alkhalifah, T. [1998] Acoustic approximations for processing in transversely isotropic media. *Geophysics*, **63**, 623–631.

Červený, V. [2001] Seismic Ray Theory. Cambridge Univ. Press.

Dellinger, J. and Symes,W. [1997] Anisotropic finite-difference traveltimes using a Hamilton-Jacobi solver. *67th SEG Annual International Meeting*, Expanded Abstracts, 1786–1789.

Faria, E. and Stoffa, P. [1994] Traveltime computation in transversely isotropic media. *Geophysics*, **59**, 272-281

Kim, S., and Cook, R. [1999] 3-D traveltime computation using second-order ENO scheme: *Geophysics*, **64**, 1867–1876.

Kumar, D., Sen M. K., and Ferguson, R. J. [2004], Traveltime calculation and prestack depth migration in tilted transversely isotropic media: *Geophysics*, **69**, 37-44.

Qian, J. and Symes, W. [2002] Finite-difference quasi-*P* traveltimes for anisotropic media. *Geophysics*, **67**, 147-155.

Qian, J., Cheng, L. and Osher, S. [2003] Level set based Eulerian methods for multivalued traveltimes in both isotropic and anisotropic media: *73th SEG Annual International Meeting*, Expanded Abstracts , 1801-1804.

Wang, Y., Nemeth, T. and Langan, R. [2006] An expanding-wavefront method for solving the eikonal equations in general anisotropic media. *Geophysics*, **71**, T129-T135.

Zhang, L., Rector, J. and Hoversten, G. M., [2002] An Eikonal Solver in Tilted TI media. *72th SEG Annual International Meeting*, Expanded Abstracts, 1955–1958.