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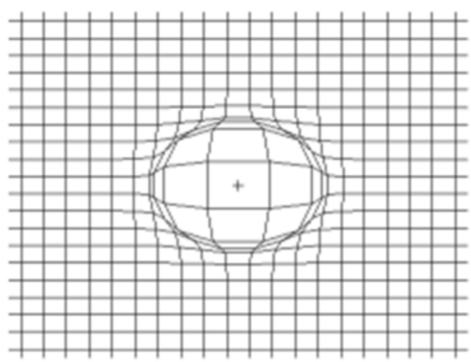
Contents

Longitudinal (compressional) wave (P-wave)

Longitudinal waves are waves that have same direction of oscillations or vibrations along or parallel to their direction of travel, which means that the oscillations of the medium (particle) is in the same direction or opposite direction as the motion of the wave. Mechanical longitudinal waves have been also referred to as **compressional waves** or **compression waves**.

Plane pressure wave



Examples of non-electromagnetic longitudinal waves include sound waves (alternation in pressure, particle displacement, or particle velocity propagated in an elastic material) and seismic P-waves (created by earthquakes and explosions).

Sound (acoustic) waves

In the case of longitudinal harmonic sound waves, the frequency and wavelength can be described with the formula

$$y(x,t) = y_0 \sin\left(\omega\left(t - \frac{x}{c}\right)\right)$$

where:

- $y(x,t)$ is the of particles from the stable position, in the direction of propagation of the wave;
- x is the displacement from the source of the wave to the point under consideration;
- t is the time elapsed;
- y_0 is the amplitude of the oscillations,
- c is the speed of the wave; and
- ω is the angular frequency of the wave.

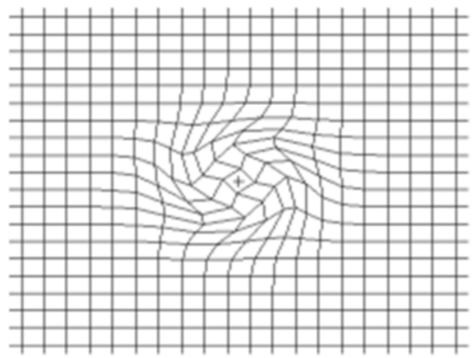
The quantity x/c is the time that the wave takes to travel the distance x.
The ordinary frequency f, in hertz, of the wave can be found using

$$f = \frac{\omega}{2\pi}.$$

For sound wave the amplitude of the wave is the difference between the pressure of the undisturbed air and the maximum pressure caused by the wave.
Sound's propagation speed depends on the type, temperature and pressure of the medium through which it propagates.

Shear wave (S-wave)

A type of seismic wave, the **S-wave**, **secondary wave**, or **shear wave** (sometimes called an **elastic S-wave**) is one of the two main types of elastic body waves, so named because they move through the body of an object, unlike surface waves.



The S-wave move as a shear or transverse wave, so motion is perpendicular to the direction of wave propagation: S-waves, like waves in a rope, as opposed to waves moving through a slinky, the P-wave. The wave moves through elastic media, and the main restoring force comes from shear effects. These waves are divergenceless and obey the continuity equation for incompressible media:

$$\nabla \cdot \mathbf{u} = 0$$

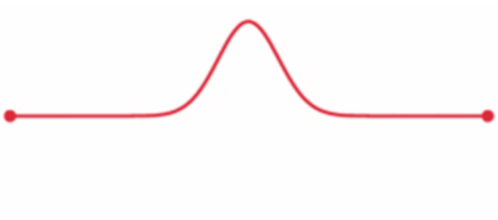
Its name, S for secondary, comes from the fact that it is the second direct arrival on an earthquake seismogram, after the compressional primary wave, or P-wave, because S-waves travel slower in rock. Unlike the P-wave, the S-wave cannot travel through the molten outer core of the Earth, and this causes a shadow zone for S-waves opposite to where they originate. They can still appear in the solid inner core: when a P-wave strikes the boundary of molten and solid cores, called the Lehmann discontinuity, S-waves will then propagate in the solid medium. And when the S-waves hit the boundary again they will in turn create P-waves. In fact, this property allows seismologists to determine the nature of the inner core.
The velocity of an S-wave in an isotropic medium can be described by the shear modulus μ and density ρ.

$$v_S = \sqrt{\frac{\mu}{\rho}}$$

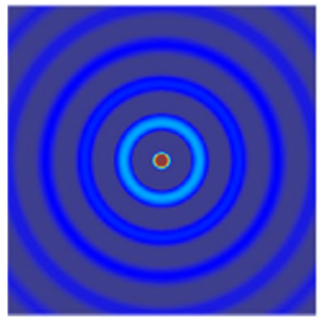
As transverse waves, S-waves exhibit properties, such as polarization and birefringence, much like other transverse waves. S-waves polarized in the horizontal plane are classified as SH-waves. If polarized in the vertical plane, they are classified as SV-waves. When an S- or P-wave strikes an interface at an angle other than 90 degrees, a phenomenon known as mode conversion occurs. As described above, if the interface is between a solid and liquid, S becomes P or vice versa. However, even if the interface is between two solid media, mode conversion results. If a P-wave strikes an interface, four propagation modes may result: reflected and transmitted P and reflected and transmitted SV. Similarly, if an SV-wave strikes an interface, the same four modes occur in different proportions. The exact amplitudes of all these waves are described by the Zoeppritz equations, which in turn are solutions to the wave equation.

Wave equation

The **wave equation** is an important second-order linear partial differential equation that describes the propagation of a variety of waves, such as sound waves, light waves and water waves. It arises in fields such as acoustics, electromagnetics, and fluid dynamics. Historically, the problem of a vibrating string such as that of a musical instrument was studied by Jean le Rond d'Alembert, Leonhard Euler, Daniel Bernoulli, and Joseph-Louis Lagrange.



A pulse traveling through a string with fixed endpoints as modeled by the wave equation.



Spherical waves coming from a point source.

Introduction

The wave equation is the prototypical example of a hyperbolic partial differential equation. In its simplest form, the wave equation refers to a scalar function *u* that satisfies:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u,$$

where $\nabla^2$is the Laplacian and where *c* is a fixed constant equal to the propagation speed of the wave. For a sound wave in air at 20°C this constant is about 343 m/s (see speed of sound). For the vibration of a string the speed can vary widely, depending upon the linear density of the string and the tension on it. For a spiral spring (a slinky) it can be as slow as a meter per second. More realistic differential equations for waves allow for the speed of wave propagation to vary with the frequency of the wave, a phenomenon known as dispersion. In such a case, *c* must be replaced by the phase velocity:

$$v_\mathrm{p} = \frac{\omega}{k}.$$

Another common correction in realistic systems is that the speed can also depend on the amplitude of the wave, leading to a nonlinear wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c(u)^2 \nabla^2 u$$

Also note that a wave may be superimposed onto another movement (for instance sound propagation in a moving medium like a gas flow). In that case the scalar *u* will contain a Mach factor (which is positive for the wave moving along the flow and negative for the reflected wave).

The elastic wave equation in three dimensions describes the propagation of waves in an isotropic homogeneous elastic medium. Most solid materials are elastic, so this equation describes such phenomena as seismic waves in the Earth and ultrasonic waves used to detect flaws in materials. While linear, this equation has a more complex form than the equations given above, as it must account for both longitudinal and transverse motion:

$$\rho \ddot{\mathbf{u}} = \mathbf{f} + (\lambda + 2\mu)\nabla(\nabla \cdot \mathbf{u}) - \mu \nabla \times (\nabla \times \mathbf{u})$$

where:

λ and μ are the so-called Lamé parameters describing the elastic properties of the medium,
ρ is density,
$\mathbf{f}$ is the source function (driving force),
and $\mathbf{u}$ is displacement.
Note that in this equation, both force and displacement are vector quantities. Thus, this equation is sometimes known as the vector wave equation.
Variations of the wave equation are also found in quantum mechanics and general relativity.

Poisson's ratio

Poisson's ratio (***v***), named after Siméon Poisson, is the ratio of the contraction or transverse strain (normal to the applied load), to the extension or axial strain (in the direction of the applied load).
When a sample cube of a material is stretched in one direction, it tends to contract (or occasionally, expand) in the other two directions perpendicular to the direction of stretch. Conversely, when a sample of material is compressed in one direction, it tends to expand (or rarely, contract) in the other two directions. This phenomenon is called the **Poisson effect**. Poisson's ratio (v) is a measure of the Poisson effect.

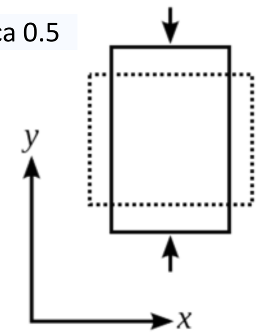


Figure 1: Rectangular specimen subject to compression, with Poisson's ratio circa 0.5

The Poisson's ratio of a stable, isotropic, linear elastic material cannot be less than −1.0 nor greater than 0.5 due to the requirement that the elastic modulus, the shear modulus and bulk modulus have positive values [1]. Most materials have Poisson's ratio values ranging between 0.0 and 0.5. A perfectly incompressible material deformed elastically at small strains would have a Poisson's ratio of exactly 0.5.

From www.glossary.oilfield.slb.com :

An elastic constant that is a measure of the compressibility of material perpendicular to applied stress, or the ratio of latitudinal to longitudinal strain. Poisson's ratio can be expressed in terms of properties that can be measured in the field, including velocities of P-waves and S-waves as shown below.

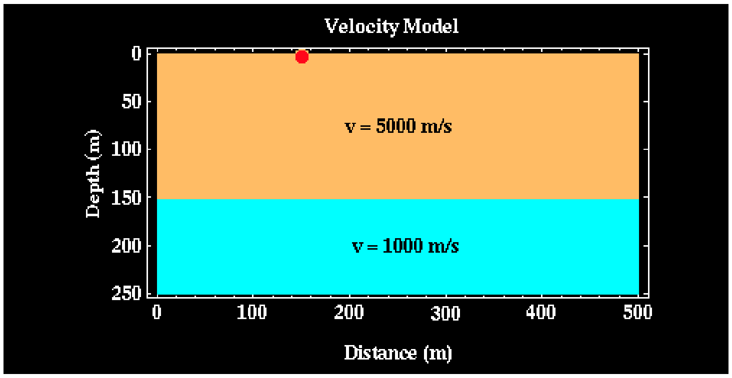
$$\sigma = {}^1\!/_2(V_P^2 - 2V_S^2)/(V_P^2 - V_S^2),$$

where σ = Poisson's ratio
V_P = P-wave velocity
V_S = S-wave velocity.

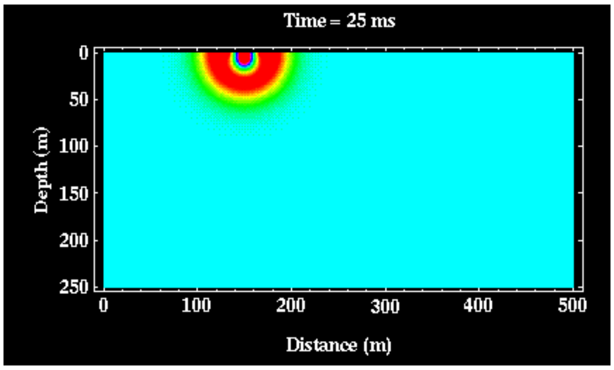
Note that if $V_S = 0$, then Poisson's ratio equals 1/2, indicating either a fluid, because shear waves do not pass through fluids, or a material that maintains constant volume regardless of stress, also known as an ideal incompressible material. V_S approaching zero is characteristic of a gas reservoir. Poisson's ratio for carbonate rocks is ~ 0.3, for sandstones ~0.2, and above 0.3 for shale. The Poisson's ratio of coal is ~ 0.4.

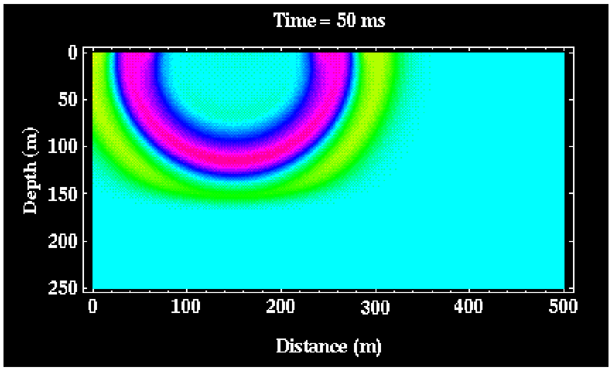
Wave Interaction with Boundaries

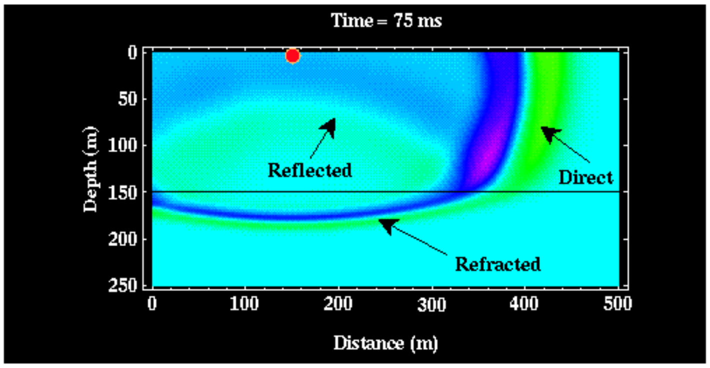
Thus far we have considered body wave propagation through media that has a constant speed of seismic wave propagation. What happens if the media consists of layers, each with a different speed of seismic wave propagation?
Consider the simple model shown below.



Although more complex than the homogeneous models considered previously, this model is still very simple, consisting of a single layer over a halfspace. In this particular example, the speed* at which seismic waves propagate in the layer is faster than the speed at which they propagate in the halfspace. Let's now watch the seismic waves propagate through this medium and see how they interact with the boundary at 150 meters. Shown below are three snapshots of the seismic wave at times of 25, 50, and 75 ms**.







From 0 to 50 ms, the wave propagates solely within the upper layer. Thus, our pictures of the wavefield look identical to those generated previously. After 50 ms, the wave begins to interact with the boundary at 150 meters depth. Part of the wave has penetrated the boundary. The portion of the wavefield that has penetrated the boundary is referred to as the *refracted wave****. Also notice that part of the wave has bounced off, or reflected off, the boundary. This part of the wavefield is referred to as the *reflected wave****. This is the portion of the wavefield that is used in reflection surveying. Finally, part of the wavefield has not interacted with the boundary at all. This part of the wavefield is called the *direct wave*.

There are several interesting features to note about the refracted arrival.

First, notice that the wavefront defining the refracted arrival is still circular, but its radius is no longer centered on the source. Geophysicists would describe this as a change in the curvature of the wavefront.

Second, notice that the apparent wavelength of the refracted arrival is much shorter than the direct arrival.

Both of these phenomena are related to the presence of the discontinuity. Remember that the period of a wave is related to its wavelength through the speed at which the wave propagates through the medium. The wavelength is equal to the speed times the period. Thus, if the period of the wave remains constant and the speed of the medium decreases, the wavelength of the wave must also decrease.

The change in curvature of the wavefront as the wave passes through the interface implies that the raypaths describing the direction of propagation of the wave change direction through the boundary. This change in direction of the raypath as it crosses a boundary is described by a well-known law known as *Snell's Law*.

Finally, of fundamental importance to note is that if you were observing the ground's motion from any point on the Earth's surface, you would observe *two* distinct waves. Initially, you would observe an arrival that is large in amplitude and that is the direct wave. Then, some time later, you would observe a smaller amplitude reflected wave. The time difference between your observation of these two arrivals is dependent on your distance from the source, the speed of wave propagation in the layer, and the depth to the boundary. Thus, by observing this time difference we may be able to learn something about the subsurface structure.

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*Unless otherwise indicated, we will now assume that we are looking at P wave propagation through the Earth. Thus, the speeds indicated are appropriate for P waves.

**ms stands for milliseconds. One millisecond is one one-thousandth of a second.

***We have simplified the situation a bit here. In general, when a P wave interacts with a boundary, it generates not only a reflected and a refracted P wave, but it can also generate a reflected and a refracted S wave. Conversely, S waves that interact with boundaries can generate reflected and refracted P waves. These conversions of P waves to S waves and S waves to P waves are called *mode conversions*. We will assume that no mode conversions occur. For refraction surveys, this is not a seriously flawed assumption, because again, we are considering only the time of arrival of the initial wave. P to S wave mode conversions will never be the first arrival. For reflection surveys, unless we were interested in recording S wave arrivals or mode conversions, we design our survey and choose the recording equipment to minimize their effects.

Seismic Wave Speeds and Rock Properties

$$V_p = \sqrt{\frac{(\frac{4}{3}\mu + k)}{\rho}}$$

$$V_s = \sqrt{\frac{\mu}{\rho}}$$

Before pursuing wave propagation issues any further, let's take a moment to describe how all this wave propagation stuff relates to geological structure. Variations in the speed at which seismic waves propagate through the Earth* can cause variations in seismic waves recorded at the Earth's surface. For example, we've shown that reflected waves can be generated from a planar boundary in seismic wave speed that can be recorded at the Earth's surface. How do these velocity variations relate to properties of the rocks or soils through which the waves are propagating?

It can be shown that in homogeneous**, isotropic*** media the velocities of P and S waves through the media are given by the expressions shown to the right. Where *Vp* and *Vs* are the P and S wave velocities of the medium, ρ is the density of the medium, and μ and *k* are referred to as the *shear* and *bulk* moduli of the media. Taken together, μ and *k* are also known as *elastic* parameters. The elastic parameters quantitatively describe the following physical characteristics of the medium.

Bulk Modulus - Is also known as the *incompressibility* of the medium. Imagine you have a small cube of the material making up the medium and that you subject this cube to pressure by squeezing it on all sides. If the material is not very stiff, you can image that it would be possible to squeeze the material in this cube into a smaller cube. The bulk modulus describes the ratio of the pressure applied to the cube to the amount of volume change that the cube undergoes. If *k* is very large, then the material is very stiff, meaning that it doesn't compress very much even under large pressures. If *k* is small, then a small pressure can compress the material by large amounts. For example, gases have very small incompressibilities. Solids and liquids have large incompressibilities.

Shear Modulus - The shear modulus describes how difficult it is to deform a cube of the material under an applied shearing force. For example, imagine you have a cube of material firmly cemented to a table top. Now, push on one of the top edges of the material parallel to the table top. If the material has a small shear modulus, you will be able to deform the cube in the direction you are pushing it so that the cube will take on the shape of a parallelogram. If the material has a large shear modulus, it will take a large force applied in this direction to deform the cube. Gases and fluids can not support shear forces. That is, they have shear moduli of zero. From the equations given above, notice that this implies that fluids and gases do not allow the propagation of S waves.

Any change in rock or soil property that causes ρ, μ, or *k* to change will cause seismic wave speed to change. For example, going from an unsaturated soil to a saturated soil will cause both the density and the bulk modulus to change. The bulk modulus changes because air-filled pores become filled with water. Water is much more difficult to compress than air. In fact, bulk modulus changes dominate this example. Thus, the P wave velocity changes a lot across water table while S wave velocities change very little.

Although this is a single example of how seismic velocities can change in the subsurface, you can imagine many other factors causing changes in velocity (such as changes in lithology, changes in cementation, changes in fluid content, changes in compaction, etc.). Thus, variations in seismic velocities offer the potential of being able to map many different subsurface features.

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*Geophysicists refer to the speed at which seismic waves propagate through the Earth as *seismic wave velocity*. Clearly, in the context of defining how fast seismic energy is transmitted through a medium, speed is a more appropriate word to use than velocity. From our introductory physics classes, recall that velocity implies not only the speed at which something is moving but also the direction in which it is moving (i.e., speed is a scalar quantity, velocity is a vector quantity). Regardless of this well-established difference in the meaning of the two terms, in geophysical jargon, the term velocity is used as a synonym for speed.

**Homogeneous media are those whose properties do not vary with position.

***Isotropic media are those whose properties at any given position do not vary with direction.

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a. anisotropy;
b. fracturing;
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