Wavefield Theory for

Multiparameter Medium *in Tesseral*

Contents

1 Medium Elastic Properties

Corresponding to this subject theory is based on concept of continuous medium. Comprehensive description of used terms and properties is provided in http://en.wikipedia.org/wiki/Elastic_modulus . From viewpoint of geoscientist is useful presenting them in terms and properties which can be derived and/or measured at application of methods it uses.

Vp (**α**) – compression wave velocity;
Vs (**β**) – shear wave velocity;
Rho (**ρ**) – density.

Shear modulus or *modulus of rigidity* (**G** or μ): **μ** = $\rho*\beta^2$
P-wave modulus M: ***M*** = $\rho*\alpha^2$

Lambda (λ) or Lame's first parameter: ***λ*** *= ρ*(α^2+2β^2)*

Bulk modulus K: ***K*** *= M-(4/3)G*

Poisson's ratio (v): ***v*** *= (α^2-2β^2)/ 2(α^2+β^2)*

2 Wave equation theory (2D case)

Equations of wave propagation in elastic medium are derived from the basic equations below:

$$\tau_{ij} = \sum_{k,l} c_{ijkl}\varepsilon_{kl}$$ - Hook law.

$$\rho \frac{\partial^2 w_i}{\partial t^2} = \sum_j \frac{\partial \tau_{ij}}{\partial x_j}$$ - Newton's second law.

$$\varepsilon_{kl} = \varepsilon_{lk} = \frac{1}{2}\left(\frac{\partial w_k}{\partial x_l} + \frac{\partial w_l}{\partial x_k}\right)$$ - strain tensor and its relationship with displacement.

$$\tau_{ij} = \tau_{ji}$$ - stress tensor.

$$c_{ijkl} = c_{klij} = c_{jikl} = c_{ijlk}$$ - 4-order symmetric elasticity tensor.

w_i - is the displacement of medium particles.

Velocity-stress wave equations for elastic medium are derived from equations above by converting displacements to velocities:

$$\frac{\partial \tau_{ij}}{\partial t} = \sum_{k,l} c_{ijkl} \frac{\partial u_k}{\partial x_l},$$

$$\rho \frac{\partial u_i}{\partial t} = \sum_j \frac{\partial \tau_{ij}}{\partial x_j},$$

where u_i is the vector of the displacement velocity.

Elasticity tensor c_{ijkl} has 81 components. But, because of its symmetry, only 21 components are independent. Tensor c_{ijkl} can be conveniently described by symmetric 6x6 matrix a_{mn}.

Conventionally the relationship between the indices (m or n) of 6x6 symmetric matrix a_{mn} and pairs of the indices (i,j) or (k,l) of 4-order tensor c_{ijkl} is denoted as: 1↔11, 2↔22, 3↔33, 4↔23, 5↔13, 6↔12.

For isotropic medium, elasticity matrix a_{mn} has notation

$$a_{mn} = \begin{pmatrix} \lambda+2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2\mu & \lambda) & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{pmatrix}$$

In Tesseral package, wavefield calculations are implemented basing on different formal approximations to physical medium.

2.1 Scalar Modeling

In this case, the physical properties of the medium are described by space-varying velocity of compression (acoustic) wave $v = v(x_1, x_3)$ in the XZ-plane and the wave field is described by displacement velocity vector $\mathbf{u} = (u_1, u_3)$ and pressure p.

This approximation of physical medium corresponds to the propagation of acoustic waves in the medium with constant density (normally, density *ρ* is assumed to 1) and the shear-wave velocity *Vs* is assumed to be 0, i.e., the case of ideal liquid with constant density.

The wavefield in the ideal liquid with constant density can be described by the system of differential equations:

$$\frac{\partial u_1}{\partial t} = \frac{\partial p}{\partial x_1}, \tag{1}$$

$$\frac{\partial u_3}{\partial t} = \frac{\partial p}{\partial x_3}, \tag{2}$$

$$\frac{\partial p}{\partial t} = v^2 \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3} \right). \tag{3}$$

In conventional way, the scalar equation is given by pressure (stress) and divergence (dilatation), which measures the expansion or compression of local medium volume.

Let's differentiate equation 1 with respect to x_1 , differentiate equation 2 with respect to x_3, and differentiate equation 3 with respect to *t. Then* we obtain:

$$\frac{\partial^2 u_1}{\partial t \partial x_1} = \frac{\partial^2 p}{\partial x_1^2} \tag{4}$$

$$\frac{\partial^2 u_3}{\partial t \partial x_3} = \frac{\partial^2 p}{\partial^2 x_3} \tag{5}$$

$$\frac{\partial^2 p}{\partial t^2} = v^2 \left(\frac{\partial^2 u_1}{\partial t \partial x_1} + \frac{\partial^2 u_3}{\partial t \partial x_3} \right) \tag{6}$$

By substituting equation (4) and (5) into (6), we obtain conventional expression of scalar equation in terms of pressure

$$\frac{1}{v^2}\frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial {x_1}^2} + \frac{\partial^2 p}{\partial {x_3}^2} \tag{7}$$

Taking into account that from equation (3) we can obtain:

$$p = v^2\left(\frac{\partial w_1}{\partial x_1} + \frac{\partial w_3}{\partial x_3}\right) = v^2 \mathrm{div}(\vec{\mathbf{w}}) = v^2\theta$$

where $\vec{\mathbf{w}}$ is the displacements vector of medium particles, and θ is divergence measuring the increasing/decreasing of local medium volume. For constant v, we obtain:

$$\frac{1}{v^2}\frac{\partial^2 \theta}{\partial t^2} = \frac{\partial^2 \theta}{\partial x_1} + \frac{\partial^2 \theta}{\partial x_3} \tag{8}$$

This is the notation of scalar equation in terms of divergence. If $v \neq \mathrm{const}$, $v^2\theta$ must be more complex function, but not θ. This means that equation (8) differs from equation (7) for pressure. Let's re-write scalar equation in form of particles displacement. To this end, let's differentiate equation (1) and equation (2) with respect to t. We obtain:

$$\frac{\partial^2 u_1}{\partial t^2} = \frac{\partial^2 p}{\partial x_1 \partial t} \tag{9}$$

$$\frac{\partial^2 u_3}{\partial t^2} = \frac{\partial^2 p}{\partial x_3 \partial t} \tag{10}$$

Let's differentiate equation (3) by x_1

$$\frac{\partial^2 p}{\partial t \partial x_1} = \frac{\partial v^2}{\partial x_1}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) + v^2\left(\frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_3}{\partial x_3 \partial x_1}\right) \tag{11}$$

Let's differentiate equation (3) by x_3

$$\frac{\partial^2 p^1}{\partial t \partial x_3} = \frac{\partial v^2}{\partial x_3}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) + v^2\left(\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + \frac{\partial^2 u_3}{\partial {x_3}^2}\right) \tag{12}$$

Combining equations (9), (11) and (10), (12), we obtain

$$\frac{\partial^2 u_1}{\partial t^2} = \frac{\partial v^2}{\partial x_1}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) + v^2\left(\frac{\partial^2 u_1}{\partial {x_1}^2} + \frac{\partial^2 u_3}{\partial x_3 \partial x}\right) \tag{13}$$

$$\frac{\partial^2 u_3}{\partial t^2} = \frac{\partial v^2}{\partial x_3}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) + v^2\left(\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + \frac{\partial^2 u_3}{\partial {x_3}^2}\right) \tag{14}$$

System of equations (13) and (14), written in terms of particle velocity vector which usually are measured in the field observations, differs from equation (8), which is usually applied in wave-equation migration procedure. Equation (8) is the basis for most seismic processing procedures. Equation (13) and (14) are an approximation to the wave equation in terms of the particle displacement velocity.

2.2 Acoustic modeling

In this case, medium properties are described by 2-D compression-wave velocity $v = v(x_1, x_3)$ and density $\rho(x_1, x_3)$.

Acoustic wave equation is described by vector of displacement velocity **u** and scalar fields of pressures p by the system of differential equations:

$$\frac{\partial u_1}{\partial t} = \frac{1}{\rho}\frac{\partial p}{\partial x_1}, \qquad (15)$$

$$\frac{\partial u_3}{\partial t} = \frac{1}{\rho}\frac{\partial p}{\partial x_3}, \qquad (16)$$

$$\frac{\partial p}{\partial t} = \rho v^2\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right). \qquad (17)$$

As in case of scalar modeling (see 2.1), let's differentiate equation (15) with respect to x_1, equation (16) with respect to x_3, and equation (17) with respect to t, then we obtain:

$$\frac{\partial^2 u_1}{\partial t \partial x_1} = \frac{\partial \frac{1}{\rho}}{\partial x_1}\frac{\partial p}{\partial x_1} + \frac{1}{\rho}\frac{\partial^2 p}{\partial x_1{}^2} \qquad (18)$$

$$\frac{\partial^2 u_3}{\partial t \partial x_3} = \frac{\partial \frac{1}{\rho}}{\partial x_3}\frac{\partial p}{\partial x_3} + \frac{1}{\rho}\frac{\partial^2 p}{\partial x_3{}^2} \qquad (19)$$

$$\frac{\partial^2 p}{\partial t^2} = \rho v^2\left(\frac{\partial^2 u_1}{\partial t \partial x_1} + \frac{\partial^2 u_3}{\partial t \partial x_3}\right) \qquad (20)$$

By substituting (18) and (19) into equation (20), we obtain the acoustic equation in terms of pressure:

$$\frac{\partial^2 p}{\partial t^2} = \rho v^2\left[\left(\frac{\partial \frac{1}{\rho}}{\partial x_1}\frac{\partial p}{\partial x_1} + \frac{1}{\rho}\frac{\partial^2 p}{\partial x_1{}^2}\right) + \left(\frac{\partial \frac{1}{\rho}}{\partial x_3}\frac{\partial p}{\partial x_3} + \frac{1}{\rho}\frac{\partial^2 p}{\partial x_3}\right)\right] = v^2\left(\frac{\partial^2 p}{\partial x_1} + \frac{\partial^2 p}{\partial x_3}\right) + \rho v^2\left(\frac{\partial \frac{1}{\rho}}{\partial x_1}\frac{\partial p}{\partial x_1} + \frac{\partial \frac{1}{\rho}}{\partial x_3}\frac{\partial p}{\partial x_3}\right)$$

As seen from the equation above, spatial derivatives of density appears. If the spatial derivative of density is close to zero, then the acoustic equation is reduced to scalar equation.

2.3 Elastic isotropic modeling

Properties of isotropic elastic medium are described by 3 spatial-varying parameters: compression-wave velocity of $v_p(x_1, x_3)$, shear-wave velocity $v_s(x_1, x_3)$ and density $\rho(x_1, x_3)$.

From parameters v_p, v_s *and* ρ, Lame's parameters can be calculated as $\lambda = \rho(v_p^2 - 2v_s^2)$ and $\mu = \rho v_s^2$, which correspond to elastic constants $a_{13} = \lambda,\ a_{55} = \mu$. In case of isotropic elastic approximation, the relationship between the displacement velocity vector $\mathbf{u} = (u_1, u_3)$ and the stress tensor $\tau_{ij}(i, j = 1{,}3)$ is given by the system of differential equations:

$$\frac{\partial \tau_{11}}{\partial t} = a_{11}\frac{\partial u_1}{\partial x_1} + a_{13}\frac{\partial u_3}{\partial x_3}$$

$$\frac{\partial \tau_{33}}{\partial t} = a_{13}\frac{\partial u_1}{\partial x_1} + a_{33}\frac{\partial u_3}{\partial x_3}$$

$$\frac{\partial \tau_{13}}{\partial t} = a_{55}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) \tag{21}$$

$$\frac{\partial u_1}{\partial t} = \frac{1}{\rho}\left(\frac{\partial \tau_{11}}{\partial x_1} + \frac{\partial \tau_{13}}{\partial x_3}\right)$$

$$\frac{\partial u_3}{\partial t} = \frac{1}{\rho}\left(\frac{\partial \tau_{13}}{\partial x_1} + \frac{\partial \tau_{33}}{\partial x_3}\right)$$

2.4 Elastic anisotropic modeling

Here, the medium property is assumed to be monocline type of anisotropy, where the plane (x_1, x_3) is the symmetry plane.

The wavefield in the plane (x_1, x_3) depends only on spatial-varying elastic constants $a_{ij}(i, j = 1{,}3{,}5)$ and density ρ.

For time-domain anisotropic elastic modeling, the displacement velocity vector $\mathbf{u} = (u_1, u_3)$ and the stress tensor $\tau_{ij}(i, j = 1{,}3)$ are related by system of differential equations:

$$\frac{\partial \tau_{11}}{\partial t} = a_{11}\frac{\partial u_1}{\partial x_1} + a_{13}\frac{\partial u_3}{\partial x_3} + a_{15}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right)$$

$$\frac{\partial \tau_{33}}{\partial t} = a_{13}\frac{\partial u_1}{\partial x_1} + a_{33}\frac{\partial u_3}{\partial x_3} + a_{35}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) \tag{22}$$

$$\frac{\partial \tau_{13}}{\partial t} = a_{15}\frac{\partial u_1}{\partial x_1} + a_{35}\frac{\partial u_3}{\partial x_3} + a_{55}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) \quad \frac{\partial u_1}{\partial t} = \frac{1}{\rho}\left(\frac{\partial \tau_{11}}{\partial x_1} + \frac{\partial \tau_{13}}{\partial x_3}\right)$$

$$\frac{\partial u_3}{\partial t} = \frac{1}{\rho}\left(\frac{\partial \tau_{13}}{\partial x_1} + \frac{\partial \tau_{33}}{\partial x_3}\right)$$

The elastic constants a_{ij} of anisotropic medium are calculated based on the assumption that the medium consist of transversally-isotropic medium with inclined symmetry axis (TTI), which can include up to 3 fracturing systems, located in parallel planes.

The physical properties of TTI medium are defined with 7 parameters: ρ - density; v_p, v_s - velocities of propagation of qP and qSV waves along symmetry axis of TTI medium; $\varepsilon, \delta, \gamma$ - Thomsen's anisotropy parameters; φ - angle of inclination of symmetry axis with respect to vertical direction.

Each fracture system is described with 3 parameters: fracture intensity $\Delta_n^{(i)}$, $\Delta_t^{(i)}$ (normal and tangential weakness) and angles of inclination with respect to vertical direction ψ_i ($i \le 3$).

Computation of coefficients of total elastic matrix $a_{ij}\,(i, j = 1,3,5)$ is done in a few steps.

For transversally isotropic medium with vertical symmetry (VTI), the elasticity matrix is given as

$$a_{33} = \rho\, v_P^2\,,$$

$$a_{44} = a_{55} = \rho\, v_S^2\,,$$

$$a_{11} = a_{22} = a_{33} \cdot (1 + 2\varepsilon)\,,$$

$$a_{66} = a_{55} \cdot (1 + 2\gamma)\,, \tag{22}$$

$$a_{12} = a_{11} - 2a_{66}\,,$$

$$a_{13} = \sqrt{\left[\,(1 + 2\delta)\,a_{33} - a_{55}\right](a_{33} - a_{55})} - a_{55}\,,$$

$$a_{23} = a_{13}\,.$$

Let's assume the resultant elasticity matrix is

$$\mathbf{A} = (a_{ij})_{ij=1..6} \tag{23}$$

To obtain the elasticity matrix which incorporate 3 fracture systems, firstly we need to obtain the inverse of elasticity matrix A (along with inverse to a_{44} , a_{55} and a_{66} values):

$$\mathbf{C} = (c_{ij})_{ij=1..6} = \mathbf{A}^{-1}.$$

Then, the inverse matrix $\mathbf{A}^{-1} = (c_{ij})_{ij=1..6}$ is rotated by angle φ by using Bond's formula for compliance matrix. After applying the rotation, the matrix $\mathbf{F} = (f_{ij})_{ij=1..6}$ is obtained. To solve the wave propagation in the symmetry plane of monocline anisotropic medium, only the coefficients f_{ij} are needed:

$$f_{11} = \frac{1}{8}(3 + \cos 4\varphi)(c_{11} + c_{33}) + \frac{1}{2}\cos 2\varphi\,(c_{11} - c_{33}) + \frac{1}{8}(1 - \cos 4\varphi)(c_{55} + 2c_{13});$$

$$f_{33} = \frac{1}{8}(3 + \cos 4\varphi)(c_{11} + c_{33}) - \frac{1}{2}\cos 2\varphi\,(c_{11} - c_{33}) + \frac{1}{8}(1 - \cos 4\varphi)(c_{55} + 2c_{13});$$

$$f_{12} = \frac{1}{2}\cos 2\varphi\,(c_{12} - c_{23}) + \frac{1}{2}(c_{12} + c_{23});$$

$$f_{22} = c_{22};$$

$$f_{13} = \frac{1}{8}(1 - \cos 4\varphi)(c_{11} + c_{33} - c_{55}) + \frac{1}{4}(3 + \cos 4\varphi)\,c_{13};$$

$$f_{15}=\frac{1}{4}\sin 4\varphi\left(c_{11}-2c_{13}+c_{33}-c_{55}\right)+\frac{1}{2}\sin 2\varphi\left(c_{11}-c_{33}\right);$$

$$f_{23}=-\frac{1}{2}\cos 2\varphi\left(c_{12}-c_{23}\right)+\frac{1}{2}\left(c_{12}+c_{23}\right);$$

$$f_{25}=\sin 2\varphi\left(c_{12}-c_{23}\right);$$

$$f_{35}=-\frac{1}{4}\sin 4\varphi\left(c_{11}-2c_{13}+c_{33}-c_{55}\right)+\frac{1}{2}\sin 2\varphi\left(c_{11}-c_{33}\right)$$

$$f_{55}=\frac{1}{2}\left(1-\cos 4\varphi\right)\left(c_{11}-2c_{13}+c_{33}-c_{55}\right)+\frac{1}{2}\sin 2\varphi\left(c_{11}-c_{33}\right)+c_{55}.$$

To incorporate each fracture system into elasticity matrix, fracture intensity $\Delta_n^{(i)}$, $\Delta_t^{(i)}$ are firstly transformed into coefficients of 6x6 matrix:

$$f_{11}^{(i)}=-\frac{1}{8}\left(1-\cos 4\psi_i\right)\left(K_n^{(i)}-K_t^{(i)}\right)+\frac{1}{2}\left(1+\cos 2\psi_i\right)K_n^{(i)};$$

$$f_{33}^{(i)}=-\frac{1}{8}\left(1-\cos 4\psi_i\right)\left(K_n^{(i)}-K_t^{(i)}\right)+\frac{1}{2}\left(1-\cos 2\psi_i\right)K_n^{(i)};$$

$$f_{55}^{(i)}=\frac{1}{2}\left(1-\cos 4\psi_i\right)K_n^{(i)}+\frac{1}{2}\left(1+\cos 4\psi_i\right)K_t^{(i)};$$

$$f_{15}^{(i)}=\frac{1}{2}\sin 2\psi_i K_n^{(i)}+\frac{1}{4}\sin 4\psi_i\left(K_n^{(i)}-K_t^{(i)}\right);$$

$$f_{35}^{(i)}=\frac{1}{2}\sin 2\psi_i K_n^{(i)}-\frac{1}{4}\sin 4\psi_i\left(K_n^{(i)}-K_t^{(i)}\right);$$

$$f_{13}^{(i)}=\frac{1}{8}\left(1-\cos 4\psi_i\right)\left(K_n^{(i)}-K_t^{(i)}\right).$$

Where $K_n^{(i)}=\dfrac{\Delta_n^{(i)}}{a_{11}(1-\Delta_n^{(i)})}$ and $K_t^{(i)}=\dfrac{\Delta_t^{(i)}}{a_{44}(1-\Delta_t^{(i)})}$.

Then, each of obtained matrixes $\mathbf{F}_i$ is added into matrix **F**. The resultant matrix $\mathbf{G}=\mathbf{F}+\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3$ is inverted, i.e., $\mathbf{A}=\mathbf{G}^{-1}$. The values of elements of inverse matrix are then used as the coefficients in the differential equations, which describe the wave propagation in anisotropic medium with fracture systems.

For the medium with only one system of fracture whose plane is perpendicular to the OX axis, the elasticity matrix has the following form:

$$\mathbf{A}=\begin{pmatrix} a_{11}(1-\Delta_n) & a_{12}(1-\Delta_n) & a_{13}(1-\Delta_n) & 0 & 0 & 0 \\ a_{12}(1-\Delta_n) & a_{22}(1-\zeta^2\Delta_n) & a_{23}(1-\zeta\Delta_n) & 0 & 0 & 0 \\ a_{13}(1-\Delta_n) & a_{23}(1-\zeta\Delta_n) & a_{33}(1-\zeta^2\Delta_n) & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55}(1-\Delta_t) & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66}(1-\Delta_t) \end{pmatrix} \qquad (27)$$

where a_{ij} is the elasticity coefficients of surrounding medium, and $\zeta = a_{12}/a_{11}$.

For the medium with monocline type of anisotropy with X-Z symmetry plane, Elasticity matrix has the form:

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 & a_{15} & 0 \\ a_{12} & a_{22} & a_{23} & 0 & a_{25} & 0 \\ a_{13} & a_{23} & a_{33} & 0 & a_{35} & 0 \\ 0 & 0 & 0 & a_{44} & 0 & a_{46} \\ a_{15} & a_{25} & a_{35} & 0 & a_{55} & 0 \\ 0 & 0 & 0 & a_{46} & 0 & a_{66} \end{pmatrix} \tag{28}$$

Parameters Δ_n and Δ_t have different values for fractured medium with gas-saturated or fluid-saturated pore.

Let's denote $g = \frac{c_{55}}{c_{33}} = \frac{V_S^{\,2}}{V_P^{\,2}}$, and *e* is the fracturing density in the background medium, then for fractured medium with gas-saturated pores:

$$\Delta_n = \frac{4e}{3g(1-g)}, \tag{29}$$

$$\Delta_t = \frac{16e}{3(3-2g)}.$$

For medium with fluid-saturated fractures:

$$\Delta_n = 0, \tag{30}$$

$$\Delta_t = \frac{16e}{3(3-2g)}.$$

Presence of fractures leads to seismic anisotropy, whose symmetry axis is normal to the plane of fractures. If background medium is isotropic, then one can determine Thomsen's parameters by formula:

$$\varepsilon = -2g(1-g)\Delta_n , \tag{31}$$

$$\delta = -2g[(1-2g)\Delta_n + \Delta_t],$$

$$\gamma = -\frac{\Delta_n}{2},$$

$$\eta = 2g(\Delta_t - g\Delta_n) .$$

For Hudson's model, one can compute Thomsen's parameters for gas-saturated pores:

$$\varepsilon = -\frac{8}{3}e \tag{32}$$

$$\delta = -\frac{8}{3}e\left[1 + \frac{g(1-2g)}{(3-2g)(1-g)}\right],$$

$$\gamma = -\frac{8e}{3(3-2g)},$$

$$\eta = \frac{8}{3}e\left[\frac{g(1-2g)}{(3-2g)(1-g)}\right].$$

$$\gamma = -\frac{8e}{3(3-g)}.$$

And if pores are filled with fluid, then

$$\varepsilon = 0; \qquad (33)$$

$$\delta = -\frac{32e}{3(3-2g)};$$

$$\gamma = -\frac{8e}{3(3-2g)}.$$

Those formula can be resolved to obtain Δ_n and Δ_t, and thus be used for determining the intensity of fracturing from Thomsen's parameters.

2.5 Connection of anisotropy parameters and elastic properties

In ***Tesseral*** package, each layer usually represents homogeneous anisotropic medium.

Generally, anisotropy is monoclinal with symmetry axis coinciding with the plane of computations. The anisotropic parameters are taken into account in the ***Anisotropic wave-equation approximation***.

For fracturing systems in the transversally isotropic medium (TI), the symmetry axis of TI-medium or the normal to planes of fracturing are assumed to be within the computation plane.

There is a detailed explanation of the parameter Dn and Dt is explained in the article of Bakulin, Grechko and Tsvankin: Estimation of fracture parameters from reflection seismic data', Geophysics, 2000, 65, N6, 1788-1830". In this article, the authors are showing additional literature. Just to keep it brief, the parameters of Dn, Dt ("weakening") are showing the level of the "sliding" along the fractures. The larger these parameters are, the more intense the fracturing.

The alpha angle is determining the tilt of the planes of the fracturing system, for example:

Alpha = 0 degrees: the plane of the fractures are parallel to YZ plan. This is vertical fracturing.
Alpha = 90 degrees: the plane of the fractures is parallel to XY plane. This is the horizontal fracturing

The Dn, Dt parameters are defined in the interface through percentages, so in order to define Dn=0.4, you need to get the number 40 in the parameters window.

In Tesseral 2D, we are looking only at plane problem – meaning that the planes of all systems are perpendicular to the calculations of XZ. However, in Tesseral 2.5D the modeling can be defined with any spatial tilt. It is done by choosing the parameters in the "3D extension" dialog in standard

Tesseral 2D. Then, there is an option to define the azimuth of fracturing in spherical system of co-ordinates on the Azimuth window.

The orthorhombic anisotropy can be formed by different methods, for example:

1) In isotropic media (in this case, the values of the epsilon, delta and gamma are equal to 0!) , we can set two or three mutually-orthogonal fracturing systems using Dn, Dt, Alpha and Azimuth parameters for each fracture system, or
2) VTI media (which is defined the anisotropy using epsilon, delta and gamma) to locate two mutually orthogonal vertical fracturing systems – defining them through epsilon, delta and gamma). Inside this VTI media, we can locate two mutually orthogonal vertical fractured systems defined as Dn, Dt, Alpha and Azimuth

In any case, in order to define the orthorhombic model, we need to use the dialog with fracturing. The Thomsons parameters are related only to the containing media. After the fracturing is entered you cannot – most of the time – describe this model with **regular** Thomson's parameters.

Sometimes, the Thomsons parameters can be generalized for a more generic scenario – but we do not do this. Neither weak, nor strong anisotropy – if it is not a TI anisotropy – can not be described using regular Thomson's parameters.

On other hand, if the anisotropy is a TI anisotropy, then - regardless of how strong it is – it can be described by Thomsons parameters exactly.

In order to use the parameters from the elasticity matrix directly, they either need to be re-calculated in the parameters we are using, or to modify Tesseral or / and Tesseral 2,5D so the elasticity matrix parameters can be imported directly. It was already discussed previously.

Anisotropy parameters are entered in 2 steps:

1) TI-medium anisotropy is described by entering Thomsen's coefficients ε, δ, γ and angle ϕ of symmetry axis with respect to the vertical direction.
2) For each fracturing system, users then enter the parameters Δ_n and Δ_t (no units) and inclination angle of fracture plane (ϕ) with respect to the vertical direction.

In *Tesseral* package, users can add up to 3 fracturing systems into the background isotropic or TI-medium with tilted symmetry axis.

As known, non-dimensional Thomsen's parameters are defined according to formulas below:

$$\varepsilon = \frac{c_{11} - c_{33}}{2c_{33}}, \qquad \delta = \frac{(c_{13} + c_{44})^2 - (c_{33} - c_{44})^2}{2c_{33}(c_{33} - c_{44})}, \qquad \gamma = \frac{c_{66} - c_{44}}{2c_{44}},$$

where C_{ij} is symmetric 6x6 matrix, containing the coefficients of elasticity tensor.

Thomsen's parameters can be found by core measurements or in literature, or obtained by measuring kinematic parameters of wave field $V_{qP}(\theta)$, $V_{qSV}(\theta)$, $V_{qSH}(\theta)$, $V_{CDP}(\varphi)$. Generally, Thomsen's parameters vary from -0.5 to 0.5.

Presence of fracturing system in parallel planes, perpendicular to the computation plane is changing the anisotropy model. In general case, it becomes monoclinal. Parameters Δ_n and Δ_t characterize the intensity of fracturing and thus influence the kinematic and dynamic properties of the wave field. Parameters Δ_n and Δ_t depend on the fractures density and material filling the pores.

Let's denote $g = \frac{V_{qSV}^2}{V_{qP}^2}$ and e is the fracturing density, which equal to the average volume of pore space on a unit of the rock volume.

Parameter Δ_t can be determined from formula

$$\Delta_t = \frac{16e}{3(3-2g)},$$

And Δ_n depend on the material of filling the pores. If it is gas, then

$$\Delta_n = \frac{4e}{3g(1-g)},$$

If it is a fluid, then Δ_n=0.

Presence of vertical fracturing system in isotropic background medium, leads to seismic anisotropy, and medium becomes horizontally-isotropic (HTI) with Thomsen's parameters depending on the material filling the pores.

If pores are filled with gas, then

$$\varepsilon = -\frac{8e}{3}.$$

$$\delta = -\frac{8e}{3}\left[1 + \frac{g(1-2g)}{(3-2g)(1-g)}\right].$$

$$\gamma = -\frac{8e}{3(3-g)}.$$

And if poses are filled with fluid, then

$$\varepsilon = 0.$$

$$\delta = -\frac{32e}{3(3-2g)}.$$

$$\gamma = -\frac{8e}{3(3-2g)}.$$

There are different ways of linking Thomsen's parameters and fracture system with the parameters used in wave equation.

Many ***references*** can be found in the papers of A. Bakulin, V. Grechko, I. Svankin:

> Estimation of fracture parameters from reflection data - Part 1,2,3. GEOPHYSICS, VOL.65, NO.6 (2000) p.1788-1830.

3 Using Q-factor at Energy attenuation estimations

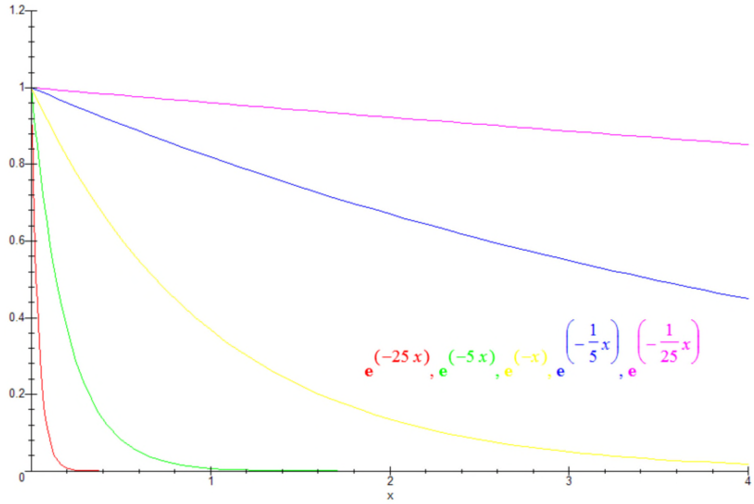
*Absorption decrement **A*** measure the attenuation of the wave on one wavelength. The quality factor can be expressed as Q=1/**A** (smaller Quality factor means stronger absorption).

Quality factor Q can also be defined as

$Q=F_0/2\alpha$,

where F_0 is frequency of the signal and α is the attenuation parameter and represents the rate of *exponential decay* (a quantity is said to be subject to exponential decay if it decreases at a rate proportional to its value) of the *wave energy* (e.g., after an *initial impulse*). A higher quality factor means a smaller attenuation.

The free encyclopedia http://en.wikipedia.org/wiki/Q_factor



Exponential decay means that a quantity decreases at a rate proportional to its value. Mathematically, this can be expressed as the following differential equation, where *N* is the quantity and λ is a positive number called the decay constant.

$$\frac{dN}{dt} = -\lambda N.$$

The solution to this equation (see below for derivation) is:

$$N(t) = N_0 e^{-\lambda t}.$$

Here *N(t)* is the quantity at time *t*, and $N_0 = N(0)$ is the initial quantity at time *t = 0*.

3.1 Solution of the differential equation of exponential decay

The equation that describes exponential decay is

$$\frac{dN(t)}{dt} = -\lambda N(t)$$

After re-arranging,

$$\frac{dN(t)}{N(t)} = -\lambda dt.$$

After integrating, we have

$$\ln N(t) = -\lambda t + C$$

where C is the *integration constant*, and hence

$$N(t) = e^{C} e^{-\lambda t} = N_0 e^{-\lambda t}$$

where $N_0 = e^C$ is obtained by evaluating the equation at $t = 0$, as N_0 is the quantity at $t = 0$.

This equation is commonly used to describe exponential decay. Any the decay constant λ is sufficient to characterize the decay. The notation λ for the decay constant is a remnant of the usual notation for an *eigenvalue*. In this case, λ is the eigenvalue of the opposite of the differentiation operator with $N(t)$ as the corresponding *eigenfunction*. The unit of decay constant is s^{-1}.

Note: In general case, quality factor Q depends on the frequency content of the seismic wave propagating in visco-elastic medium, which is simulated using corresponding visco-elastic wave-equation approximation. Quality factor Q affect the velocity dispersion (wave is scattered or distorted as function of frequency). Usually, it is assumed that frequency band is narrow (for the wavelet's frequency band used in the numerical modeling is narrow). And under this assumption, the attenuation effect is taken into account by using the quality factor at peak frequency F_0 of the source wavelet. It may be called as ***frequency-band independent or Fp-approximation of energy attenuation***.

For some particular modeling tasks such approximation may be considered as too simplifying this wave propagation phenomenon, when there is need to study influence of the medium absorption at deeper level of modeling of *frequency dependent influence of attenuating properties* (Q-factor parameter) of the medium. The ***visco-elastic approximation*** allows to model such complex effects of wave propagation as frequency dependent wave attenuation and velocity dispersion caused by absorbing properties of the medium.

4 Modeling wave fields in 2D visco-elastic isotropic medium

Note: The numerical computation of seismic wavefield in linear viscoelastic media is complicated by the existence of convolution integrals in the governing equations. The problem can be solved by approximating each continuous relaxation spectrum by a discrete one, whose corresponding complex modulus is a rational function of frequency. The convolution integrals can then be eliminated by introducing a sequence of variables, with each satisfying a first order differential equation in time (Day and Minster, 1984; Emmerlich and Korn, 1987; Carcione et al., 1988). The resulting system of governing differential equations can then be solved numerically in various ways.

During wave propagation in a real geological medium, energy loss is caused by internal friction. Intensity of energy loss is characterized by Q value. For a plane wave with frequency ω propagating along x direction with velocity v, the amplitude of this plane wave can be calculated as:

$$a(x) = a(0)\exp\left[-\frac{\omega t}{2Q}\right] = a(0)\exp\left[-\frac{\omega x}{2vQ}\right],$$

Where $x = vt$, $a(x)$ is the wave amplitude in location x or at time t.

This formula means that, after propagating over a period $T=\frac{2\pi}{\omega}$ or a wavelength $\lambda=\frac{2\pi v}{\omega}$, amplitude of plane wave is attenuated by $\exp\left[-\frac{\pi}{Q}\right]$ times.

Rock physics study shows that in broad frequency range such as $10^{-3}-10^{2}$ Hz, Q value practically is constant.

Calculation of wave field for 2D absorbing isotropic medium is based on solving following differential equation by finite-difference method

$$\rho\frac{\partial v_1}{\partial t}=\frac{\partial \delta_{11}}{\partial x_1}+\frac{\partial \delta_{13}}{\partial x_3}.$$

$$\rho\frac{\partial v_3}{\partial t}=\frac{\partial \delta_{13}}{\partial x_1}+\frac{\partial \delta_{33}}{\partial x_3}.$$

$$\frac{\partial \delta_{11}}{\partial t}=\tilde{\tilde{\pi}}\left(\frac{\partial v_1}{\partial x_1}+\frac{\partial v_3}{\partial x_3}\right)-2\tilde{\tilde{\mu}}\frac{\partial v_3}{\partial x_3}+\sum_{l=1}^{L}r_{11}^{l}$$

$$\frac{\partial \delta_{13}}{\partial t}=\tilde{\tilde{\mu}}\left(\frac{\partial v_1}{\partial x_3}+\frac{\partial v_3}{\partial x_1}\right)+\sum_{l=1}^{L}r_{13}^{l}$$

$$\frac{\partial \delta_{33}}{\partial t}=\tilde{\tilde{\pi}}\left(\frac{\partial v_1}{\partial x_1}+\frac{\partial v_3}{\partial x_3}\right)-2\tilde{\tilde{\mu}}\frac{\partial v_1}{\partial x_1}+\sum_{l=1}^{L}r_{33}^{l}$$

where

$$\pi=\rho V_p^2,\ \mu=\rho V_s^2,\ \tilde{\tilde{\pi}}=\pi\left(\sum_{l=1}^{L}\frac{\tau_{\varepsilon l}^{P}}{\tau_{\delta l}}-L+1\right),\ \tilde{\tilde{\mu}}=\mu\left(\sum_{l=1}^{L}\frac{\tau_{\varepsilon l}^{S}}{\tau_{\delta l}}-L+1\right),$$

$\tau_{\varepsilon l}^{p}$, $\tau_{\varepsilon l}^{s}$, $\tau_{\delta l}$ are relaxation times for *l*-$_{\text{i}}$ absorption law (*l*=1,…,L).

For variables r_{ij}^{l} are used following formulas

$$\frac{\partial r_{11}^{l}}{\partial t}=-\frac{1}{\tau_{\delta l}}\left[r_{11}^{l}+\left(\tilde{\pi}^{l}-\pi\right)\left(\frac{\partial v_1}{\partial x_1}+\frac{\partial v_3}{\partial x_3}\right)-2\left(\tilde{\mu}^{l}-\mu\right)\frac{\partial v_3}{\partial x_3}\right]$$

$$\frac{\partial r_{13}^{l}}{\partial t}=-\frac{1}{\tau_{\delta l}}\left[r_{13}^{l}+\left(\tilde{\mu}^{l}-\mu\right)\left(\frac{\partial v_1}{\partial x_3}+\frac{\partial v_3}{\partial x_1}\right)\right],$$

$$\frac{\partial r_{33}^{l}}{\partial t}=-\frac{1}{\tau_{\delta l}}\left[r_{33}^{l}+\left(\tilde{\pi}^{l}-\pi\right)\left(\frac{\partial v_1}{\partial x_1}+\frac{\partial v_3}{\partial x_3}\right)-2\left(\tilde{\mu}^{l}-\mu\right)\frac{\partial v_1}{\partial x_1}\right]$$

where

$$\tilde{\pi}^{l}=\pi\frac{\tau_{\varepsilon l}^{P}}{\tau_{\delta l}},\ \tilde{\mu}^{l}=\mu\frac{\tau_{\varepsilon l}^{S}}{\tau_{\delta l}}.$$

Relaxation times $\tau_{\varepsilon l}^{p},\tau_{\varepsilon l}^{s},\tau_{\delta l}$ are tuned by the program in a way to secure independency of Q_P and Q_S with respect to frequency ω in a wide range. To run modeling with absorption, users need to input signal peak frequency f_0, values of Q_P and Q_S for each polygon and the number of absorption laws L.

Inside the program, relaxation times is automatically calculated to ensure that frequency range $\left[\omega_{beg}, \omega_{end}\right]$ will be evenly covered in the logarithmic scale by frequencies $\omega_1, \ldots, \omega_L$ and for each of them, relaxation times are calculated as

$$\tau_{\delta l} = \frac{1}{\omega_l}\left(\sqrt{1+\frac{1}{Q_{P0}^2}} - \frac{1}{Q_{P0}}\right),$$

$$\tau_{\varepsilon l}^{p} = \frac{1}{\omega_l^2 \tau_{sl}},$$

$$\tau_{\varepsilon l}^{s} = \frac{1+\omega_l \tau_{sl} Q_{S0}}{\omega_l Q_{S0} - \omega_l^2 \tau_{sl}}.$$

The program is using values $\omega_{beg} = \frac{1}{8}\omega_0$, $\omega_{end} = 8\omega_0$. ω_o is the peak frequency of the source wavelet.

Parameters Q_{P0} and Q_{S0} are consequently tuned in a least-square sense by minimizing the following objective functions ε_P and ε_S:

$$\varepsilon_P = \int_{\omega_{beg}}^{2\omega_0} \left(Q_P(\omega) - Q_P\right)^2 d\omega,$$

$$\varepsilon_S = \int_{\omega_{beg}}^{2\omega_0} \left(Q_S(\omega) - Q_S\right)^2 d\omega,$$

where

$$Q_P(\omega) = \frac{\mathrm{Re}\left(q_P(\omega)\right)}{\mathrm{Im}\left(q_P(\omega)\right)},$$

$$Q_S(\omega) = \frac{\mathrm{Re}\left(q_S(\omega)\right)}{\mathrm{Im}\left(q_S(\omega)\right)},$$

and

$$q_p(\omega) = \sum_{l=1}^{L} \frac{1+i\omega\tau_{\varepsilon l}^{p}}{1+i\omega\tau_{\delta l}} - L + 1,$$

$$q_s(\omega) = \sum_{l=1}^{L} \frac{1+i\omega\tau_{\varepsilon l}^{s}}{1+i\omega\tau_{\delta l}} - L + 1.$$

Presence of absorption causes velocity dispersion or frequency-dependant velocity $v_p(\omega)$ and $v_s(\omega)$. For ideal case $Q = const$, these functions are logarithmic

$$V_P(\omega) = V_P(\omega_{P0})\left(1+\ln\left(\frac{\omega}{\omega_{P0}}\right)\right),$$

$$V_S(\omega) = V_S(\omega_{S0})\left(1+\ln\left(\frac{\omega}{\omega_{S0}}\right)\right),$$

where ω_{p_0} and ω_{s_0} are some fixed frequencies.

The modeling is based on absorption laws

$$V_P(\omega) = V_P \,\mathrm{Re}\sqrt{q_P(\omega)}$$

$$V_S(\omega) = V_S \operatorname{Re}\sqrt{q_S(\omega)}$$

Because of $q_P(0) = q_S(0) = 1$, so $V_P(0) = V_P$, $V_S(0) = V_S$ - velocities specified in a medium model polygons.

4.1 Dependency of $Q(\omega)$ and $V(\omega)$ on angular frequency – Velocity Dispersion

For different types of waves and number of absorption laws $L = 1,3,5$, the dependency of $Q(\omega)$ and $V(\omega)$ on angular frequency is shown on the figure below, where $\omega_0 = 2\pi \cdot 40Hz$, $V_P = 2000$ m/s and $V_S = 1100$ m/s are used.

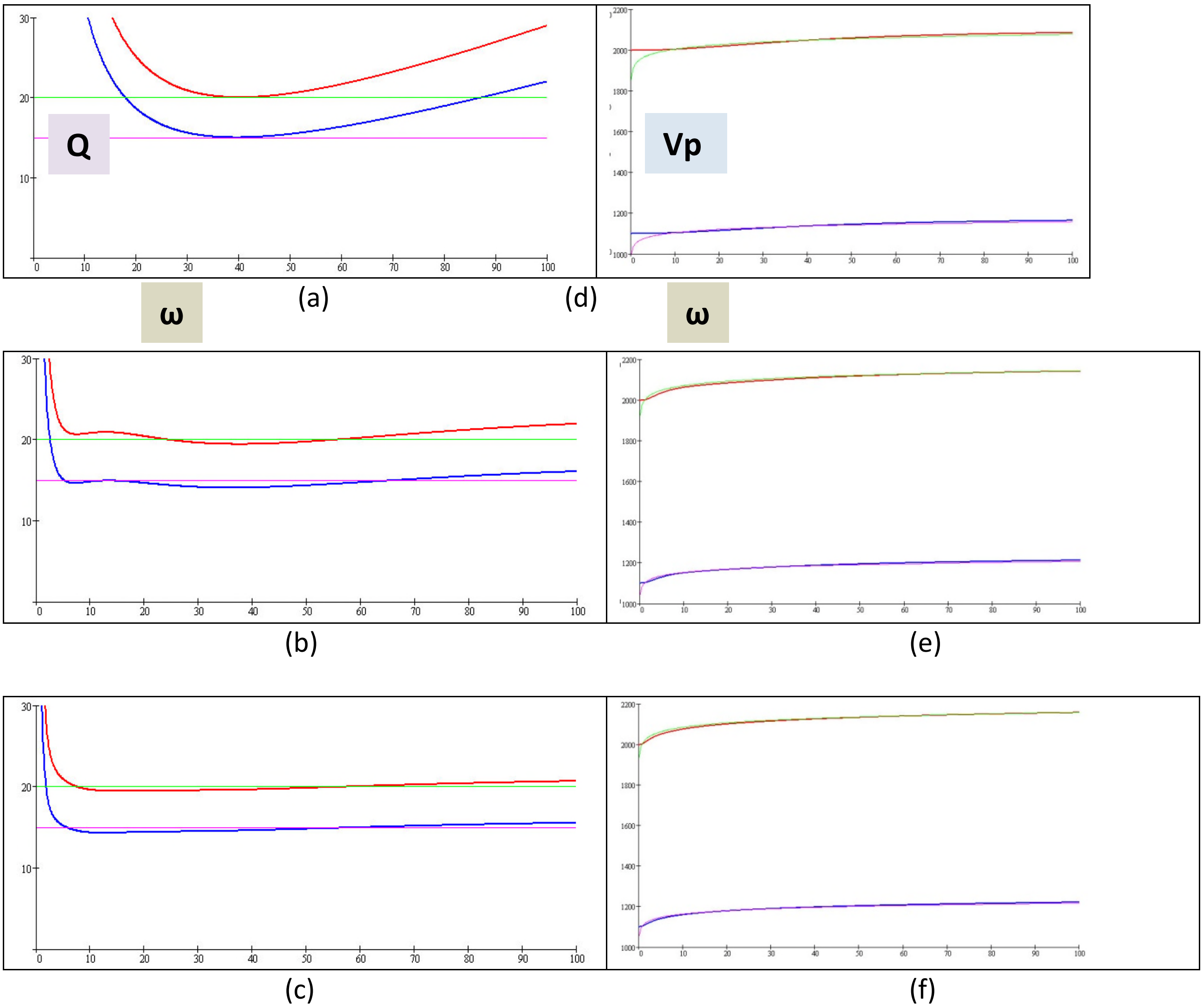


Fig.1. Left: the dependency of Q-factor values on circular frequency for primary (red) and shear (blue) waves using different number of absorption laws: (a) – 1 law; (b) -3 laws; (c) – 5 laws. **Right**: the dependency of velocity values on circular frequency for primary (red) and shear (blue) waves using different number of absorption laws: (d) – 1 law; (e) -3 laws; (f) – 5 laws. Green and pink lines show constant Q values (left) and velocity logarithmic dependency (right) for primary and shear waves.

From math physics theory can be defined relation between velocity dispersion and wave energy attenuation (Q-factor):

(1) $c(\omega) = c_1 \left[1 - \frac{1}{\pi Q} \ln(\frac{\omega}{2\pi}) \right]$

or relating to Q:

(2) $Q = c_1 \ln(\frac{\omega_1}{\omega_0}) / \pi (c(\omega_1) - c(\omega_0))$

where

$c_1 = c(\omega = 2\pi)$ - velocity of compression waves (no attenuation, frequency independent)

ω – frequency

Q – Quality factor

c(ω) – compression wave velocity at frequency ω

For example, if:

ω1=5,000 Hz

ω0=250 Hz (ω1/ω0=20)

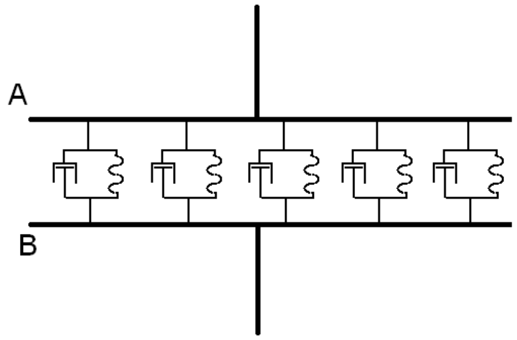
	Vp^1 ~5KHz	Vp^2 ~250Hz	ΔVp (%)	**Q**
a)	2500 m/s	2300 m/s	9%	**10**
b)	2500 m/s	2000 m/s	25%	**5**

Then for:

Case **a)**	Case **b)**
C1~=2300 m/s	C1~=2000 m/s
C(ω1)=2500 m/s	C(ω1)=2500 m/s
C(ω0)=2300 m/s	C(ω0)=2000 m/s
Q=(2300*2.9)/(3.14*200)= ~6600/630 ~=**10**	**Q**=(2500*2.9)/(3.14*500)= ~7500/1500 ~=**5**

4.2 Damping Mechanisms

Damping mechanism for viscoelastic medium can be illustrated using the picture below:



Let's press and then release plates A and B. Springs work to return to initial state, but pistons slow down this process. The *L* number of damping mechanisms (here) is *5*.

For each frequency, each defined mechanism has ***maximum relaxation time*** (in ***e*** times). Conventionally this time is called relaxation time for a given mechanism. There are 2 types of ***relaxation time***: ***Strain relaxation time*** and ***Stress relaxation time***.

Attenuation for given frequency is defined by ***Q-factor*** *(Quality factor)* which is equal to sum of attenuations of all damping mechanisms for a given frequency.

To make it working properly, it is necessary to fine-tune N (here, 5) relaxation times so that, for each frequency, Q-factor equals approximately to the defined one (as a medium property).

Stress relaxation times only can be fitted using special procedures, and the remaining ones then can be calculated using corresponding equations. Once tuned, those relaxation times can be used for all other cases.

Then, the viscoelastic wave equation for 1 damping mechanism can be extended to the case of N damping mechanisms case.

4.3 Summary

Tesseral software uses 2 methods to take into account the effect of seismic absorption:

1. The first method (***Fp-attenuation***)is to take into account amplitude attenuation by using formula $a(t) = a_0 \exp\left[-\frac{\omega_0 t}{2Q}\right]$, where ω_0 is a signal peak frequency. Implementing absorption in such way makes calculations very fast, but the signal spectrum and its shape will not change and velocity dispersion is not taken into account. This method is an approximation and it can be used as a fast way to evaluate the absorption effect, especially for conventionally used in seismic modeling signals which have narrow band of frequencies. See also chapter *Using Q-factor at Energy attenuation estimations*.
2. The second method (***visco-elastic***) takes into account mechanics of seismic absorption more accurately. It correctly calculates signal spectrum, amplitude, velocity dispersion and signal time registrations. This approach can be used for modeling the wave fields in complex geological conditions such as fracture zones. Because dry, water-saturated and oil-saturated fractures and layers have different absorption properties, it is possible to investigate their influence on the wave field.

   A value of ***L*** *(number of damping mechanisms)* has been added as one of the user input parameters in Tesseral package. It is recommended to use L=3. Time step and spatial intervals are automatically calculated by the program using the same algorithms as without absorption.

5 Gassman's model of porous medium

Comprehensive description of Gassman's equation is provided in http://en.wikipedia.org/wiki/Gassmann's_equation.

Below are presented some formulae and descriptions relating to used in Tesseral conversions from properties of the containing rock matrix (skeleton) and its liquid pore content into bulk rock properties *Vp, Vs* and *Pho*.

$$\rho = \Phi\rho_f + (1-\Phi)\rho_s$$

where ρ is the density of porous medium. ρ_f is the density of fluid, ρ_s is the density of skeleton and Φ is the porosity in percentage.

$$M = \overline{M} + \frac{\left(1-\overline{K}/K_s\right)^2}{\left[\Phi/K_f + (1-\Phi)/K_s - \overline{K}/K_s^2\right]}.$$

$\overline{M}, \overline{K}, \overline{\mu}$ is modules of planar deformation, omni-directional compression and shifting in the dry rock. M, K, μ is the modules of planar deformation, omni-directional compression and shifting in the fluid-saturated porous rock. $K = \frac{3\lambda + 2\mu}{3}$ is the module of omni-directional compression in the fluid-saturated porous rock. K_s is the module of omni-directional compression of skeleton (rock mineral).

$$M = \lambda + 2\mu;$$

$\overline{K}/K_s = (1+50\Phi)^{-1}$ is the density of porous medium.

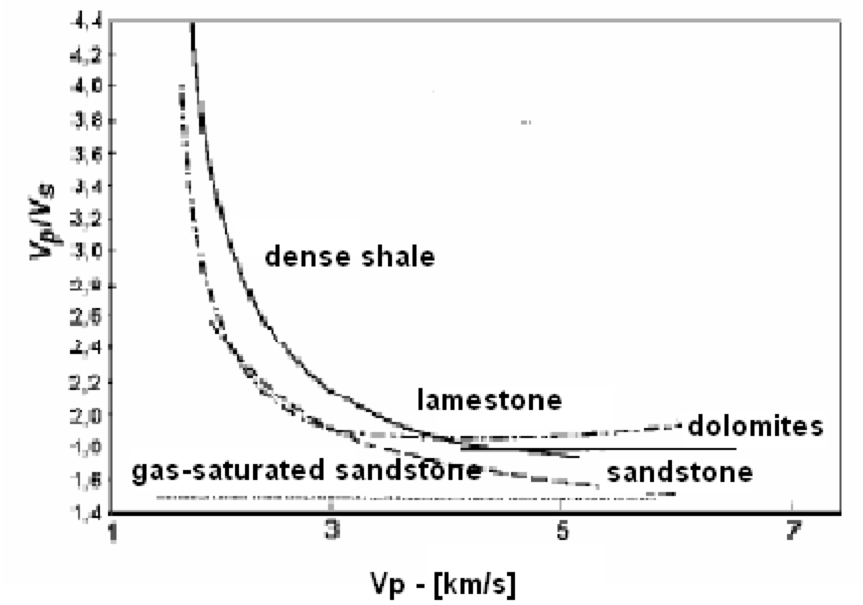


Fig.1 Dependencies VP/VS on VP for rocks of different lithology

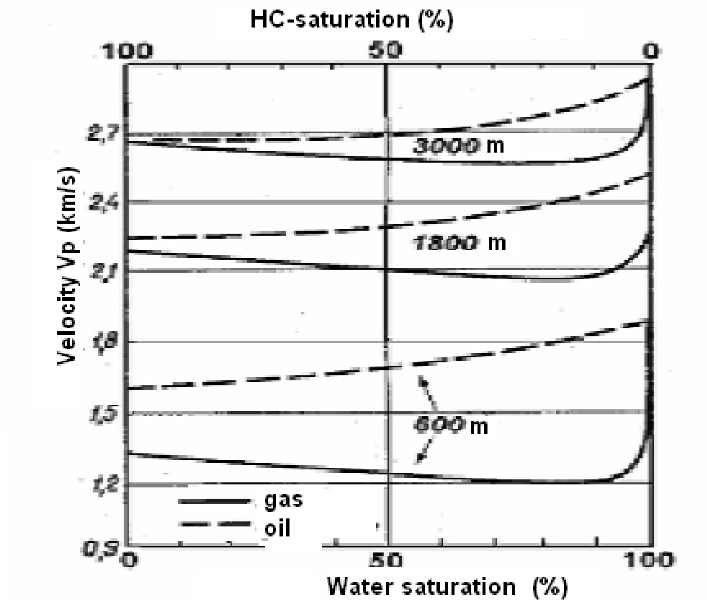


Fig.2. Velocity *VP*, as function of water-saturation coefficient, for oil- and gas-saturated sandstones (broken line) at depth of 600m, 1800m and 3000 m.
Vp=2700 m/s (gas), Vp=2900 m/s (water). Accordingly for such velocities:1/γ =1,45 (gas), 1/γ=2.0 (water). And corresponding shear velocities: Vs=1860 m/s (gas), Vs=1450 m/s

6 3D AVO modeling procedure

Matrix method is the foundation for studying the **3D AVO functionality,** which calculates the tables of reflection and transmission coefficients of interferential packs for stack of anisotropic layers with absorption, as well as their phase delays.

To characterize the reflection and refraction properties of a stack of anisotropic layers with parallel plane boundaries, particle motion in each layer is described by a system of 6 differential equations:

$$\frac{d\mathbf{f}}{dz} = j\omega\mathbf{A}\mathbf{f} \ ,$$

where $\mathbf{f} = (u_1\ u_2\ u_3\ \tau_{13}\ \tau_{23}\ \tau_{33})^T \cdot \exp(j\omega(t - p_1x_1 - p_2x_2))$ is the plane wave.

Propagation matrix $\mathbf{A}$ depends on elasticity coefficients a_{ij}, density ρ and slownesses p_1 and p_2, which determine the direction of wave propagation.

At the transmission through layers' boundaries, solution $\mathbf{f}$ remains continuous. If medium consists of stack of layers with thicknesses h_1 ,…,h_k and $\mathbf{A}_1$,…,$\mathbf{A}_k$ is the propagation matrix for each layer, then the operator, which calculates the wavefield from first boundary to the last one (propagator for a stack of layers), can be given as:

$$\mathbf{P} = \exp(j\omega\, h_k \mathbf{A}_k) \cdot \ldots \cdot \exp(j\omega\, h_1 \mathbf{A}_1)\,.$$

Let's denote $\mathbf{P} = \begin{pmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{pmatrix}$, where $\mathbf{P}_{ij}$ -is the 3x3 matrix.

Solving the system of equations $\mathbf{Pf}_1 = \mathbf{f_k}$ relatively to dissipated (outgoing from stack of layers) waves, we obtain formula for the dissipation matrix:

$$\mathbf{S} = -\begin{pmatrix} \mathbf{P}_{12} & -\mathbf{E} \\ \mathbf{P}_{22} & \mathbf{0} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{P}_{11} & \mathbf{0} \\ \mathbf{P}_{21} & -\mathbf{E} \end{pmatrix}.$$

Elements of matrix **S** are reflection and transmission coefficients of plane waves for all types of waves for the stack of layers.

In the 3D AVO module, the coefficients of reflection and transmission of plane waves from a stack of layers with horizontal boundaries are calculated by using the matrix of Haskell-Thomson method. The program forms tables of reflection and transmission coefficients depending on incidence angle and frequencies of seismic wave dissipating on those boundaries. Then, the stored data are output in form of graphs.

To calculate the dissipation matrix for a stack of layers allocated between zero and $(n+1)^m$ half-spaces, firstly the propagator matrix H for this stack is defined, which relate the amplitude of propagating waves from the first boundary to last one according to the formula $\vec{w}_{n+1} = H\vec{w}_1$, where $H = E_{n+1}^{-1} P_n \dots P_2 E_0$, and $P_i = E_i \Lambda_i E_i^{-1}$ (propagator of i^{th} layer).

The dissipation matrix S allows for calculating stack of amplitudes of six incoming waves in amplitudes of outgoing six waves. For one boundary, dissipation matrix S consists of reflection and transmission coefficients, and is defined according to the equation:

$$S = -\begin{pmatrix} h_{14} & h_{15} & h_{16} & -1 & 0 & 0 \\ h_{24} & h_{25} & h_{26} & 0 & -1 & 0 \\ h_{34} & h_{35} & h_{36} & 0 & 0 & -1 \\ h_{44} & h_{45} & h_{46} & 0 & 0 & 0 \\ h_{54} & h_{55} & h_{56} & 0 & 0 & 0 \\ h_{64} & h_{65} & h_{66} & 0 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} h_{11} & h_{12} & h_{13} & 0 & 0 & 0 \\ h_{21} & h_{22} & h_{23} & 0 & 0 & 0 \\ h_{31} & h_{32} & h_{33} & 0 & 0 & 0 \\ h_{41} & h_{42} & h_{43} & -1 & 0 & 0 \\ h_{51} & h_{52} & h_{53} & 0 & -1 & 0 \\ h_{61} & h_{62} & h_{63} & 0 & 0 & -1 \end{pmatrix}.$$

In the program, for the set range of incidence angles and frequencies, a dissipation matrixes S are determined and then the amplitudes and phases of reflection and-transmission coefficients are calculated for each boundary.

Propagator of each layer is calculated under the assumption that the medium could have TTI-anisotropy with an arbitrary inclination of a symmetry axis. Inside this medium, up to 3 fracturing systems can be added, which is characterized by the intensities and inclination angles. All these parameters are converted into elasticity coefficients. As a result, the medium generally becomes anisotropic without symmetry (i.e. triclinic). Fracturing is taken into account by using formulas by Bakulin et al., (see reference). Frequency-dependent absorption and velocity dispersion is taken into account according to formulas by Aki & Richards (see reference). Propagator matrix for an anisotropic layer is calculated on the basis of the equations given by Stroh (1962).
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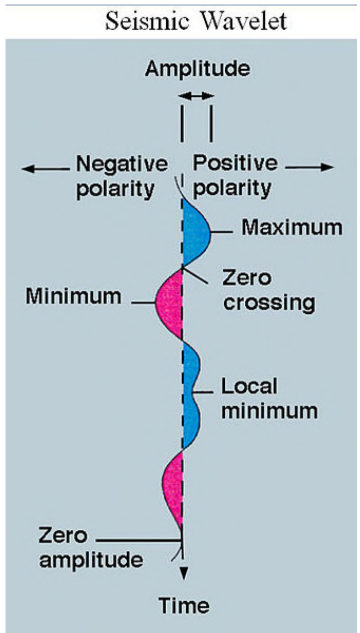
7 Wavelets and Signals

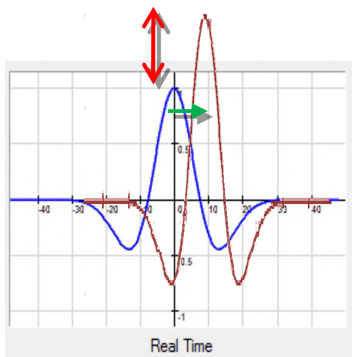


From http://en.wikipedia.org/wiki/Wavelet

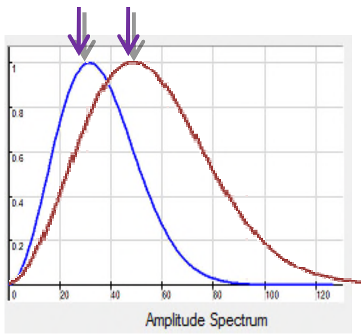
A *wavelet* is a wave-like oscillation with amplitude that begins at zero, increases, and then decreases back to zero. It can typically be visualized as a "brief oscillation" like one might see recorded by a seismograph. Generally, wavelets are purposefully crafted to have specific properties that make them useful for signal processing. Wavelets can be combined, using a "reverse, shift, multiply and integrate" technique called *convolution*, with portions of a known signal to extract information from the unknown signal.

From http://en.wikipedia.org/wiki/Seismic_source

A seismic source is a device that generates controlled *seismic energy* used to perform seismic surveys. A seismic source can be simple, such as dynamite, or it can use more sophisticated technology, such as a specialized air gun. Seismic sources can provide single pulses or continuous sweeps of energy. Both types of seismic sources generate *seismic waves*, which travel through a *medium* such as water or layers of rocks. Some of the waves then reflect and refract and is recorded by *receivers*, such as *geophones* or *hydrophones*.



There must be done additional *parameterization* in seismic (numerical) modeling for *wavelet (of particular type)* to become a *signal*. *Peak frequency (on picture - **violet arrow**)* (of the wavelet), *maximum amplitude (**red arrow**)* and *phase* (**green arrow**) must be defined.



Reference: analytical wavelets

Rikker wavelet equation:

$$f(t, f_0) = (1 - 2 \cdot \pi^2 \cdot f_0^2 \cdot t^2)\exp(-\pi^2 \cdot f_0^2 \cdot t^2)$$

Rikker wavelet Fourier transformation:

$$F_r(f, f_0) = \frac{2}{\sqrt{\pi}\, f_0}\left(\frac{f}{f_0}\right)^2 \cdot \exp\left(-\left(\frac{f}{f_0}\right)^2\right),$$

where f_0 – central signal frequency.

Puzirov wavelet equation:

$$f(t, f_0, \alpha, \varphi_0) = \exp(-\alpha^2 \cdot f_0^2 \cdot t^2)\cos(2 \cdot \pi \cdot f_0 \cdot t - \varphi_0)$$

Puzirov wavelet Fourier transformation:

$$F_p(f, f_0, \alpha, \varphi_0) = \frac{\sqrt{\pi}}{2 \cdot \alpha \cdot f_0} \cdot \left\{\exp\left(-\frac{\pi^2}{\alpha^2}\left(\frac{f}{f_0} - 1\right)^2 - i \cdot \varphi_0\right) + \exp\left(-\frac{\pi^2}{\alpha^2}\left(\frac{f}{f_0} + 1\right)^2 + i \cdot \varphi_0\right)\right\},$$

where f_0 – signal central frequency, φ_0 – phase shift of the signal relating to zero-phase, number α – determines how fast curvature of the signal is approaching zero.

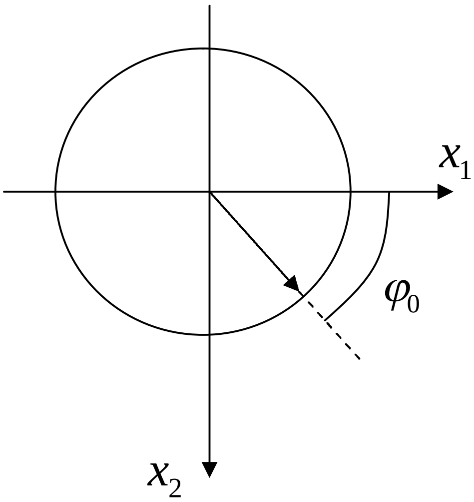
8 Complex Sources

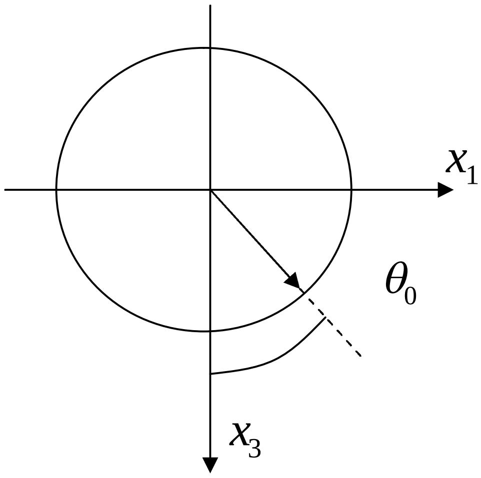
8.1 Force f of arbitrary direction

8.1.1 3D case

Vector $\mathbf{f} = (\sin\theta_0 \cos\varphi_0, \sin\theta_0 \sin\varphi_0, \cos\theta_0)$ is determined by two angles:

1) Tilt angle of the force vector $\mathbf{f}$ $0 \le \theta_0 \le 90^0$ and azimuth vector of the force vector $0 \le \varphi_0 < 360^0$ relatively to axis OX$_1$:





In the coordinate system (x_1, x_2, x_3) the force vector $\mathbf{f}$ has coordinates $\mathbf{f} = (f_1, f_2, f_3)$, where

$$\begin{cases} f_1 = \sin\theta_0 \cos\varphi_0 \\ f_2 = \sin\theta_0 \sin\varphi_0 \\ f_3 = \cos\theta_0 \end{cases}.$$

Compression P-wave and shear SV and SH-waves have direction characteristics:

$$L_p = \frac{\mathbf{f}\cdot\mathbf{r}}{4\pi\rho V_p^2}, \quad L_{SV} = \frac{\mathbf{f}\cdot\boldsymbol{\theta}}{4\pi\rho V_S^2}, \quad L_{SH} = \frac{\mathbf{f}\cdot\boldsymbol{\varphi}}{4\pi\rho V_S^2},$$

where

$\mathbf{r} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ - polarization vector of P-wave (direction to receiver),

$\boldsymbol{\theta} = (\cos\theta\cos\varphi, \cos\theta\sin\varphi, -\sin\theta)$ - polarization vector of SV-wave,

$\boldsymbol{\varphi} = (-\sin\phi, \cos\phi, 0)$ - polarization vector of SH-wave.

Vectors $\mathbf{r}$, $\boldsymbol{\theta}$ and $\boldsymbol{\varphi}$ are reciprocally perpendicular unitary vectors, at this vector $\mathbf{r}$ is directed on receiver, i.e. receiver has azimuth φ and is tilted by angle θ.

Generation of signal is executed by addition of increments to the displacement velocities

$$\Delta u_1 = s(t)\sin\theta_0\cos\varphi_0,$$
$$\Delta u_2 = s(t)\sin\theta_0\sin\varphi_0,$$
$$\Delta u_3 = s(t)\cos\theta_0.$$

where $s(t)$ - source signal.

8.1.2 2D Case

In 2D force vector $\mathbf{f}$ is determined by angle θ_0 of force to the vertical only.

In this case azimuths $\varphi = \varphi_0 = 0$ and directionality characteristics are reduced to simple formulae:

$L_p = \dfrac{\sin(\theta - \theta_0)}{4\pi\rho V_p^2}$, $L_{SV} = \dfrac{\cos(\theta - \theta_0)}{4\pi\rho V_S^2}$.

Generation of signal in the program is realized by addition of increments by time to displacement velocities:

$\Delta u_1 = s(t)\sin\theta_0$;

$\Delta u_3 = s(t)\cos\theta_0$.

8.2 Sources of coupled forces type

In 2.5D, as well as in 3D case, all kinds of couple force sources may be applicable. Those are sources, creating moments, as well as stretching and squeezing tensions. Latter are called vector dipoles.

There are possible 9 pairs of forces M_{ij} $(1 \le i, j \le 3)$. But, due to the law of conservation, pairs M_{ij} and M_{ji}, creating the same magnitude moments occur simultaneously, i.e. matrix $\mathbf{M}$ is always symmetric. Therefore, we can restrict the 6 pairs $(1 \le i \le j \le 3)$.

At continuation of the wave field in time, equations

$$\frac{\partial \tau_{mn}}{\partial t} = c_{mn,pq} \frac{\partial u_p}{\partial x_q}$$

where τ_{mn} - stresses tensor, $c_{mn,pq}$ - elasticity tensor, u_p - vector of displacement velocities. At 2.5D modeling differentiations $\dfrac{\partial u_p}{\partial x_2}$ are replaced by multiplying by the number jk_2 where k_2-spatial frequency by direction x_2, and $j = \sqrt{-1}$.

For determining of couple of forces (m_0, n_0), where $m_0 \le n_0$ formulae

$$\frac{\partial \tau_{mn}}{\partial t} = c_{mn,pq} \frac{\partial u_p}{\partial x_q}$$

at $m = m_0$ and $n = n_0$ is replaced by formulae

$$\frac{\partial \tau_{m_0 n_0}}{\partial t} = c_{m_0 n_0,pq} \frac{\partial u_p}{\partial x_q} + \frac{ds(t)}{dt}$$

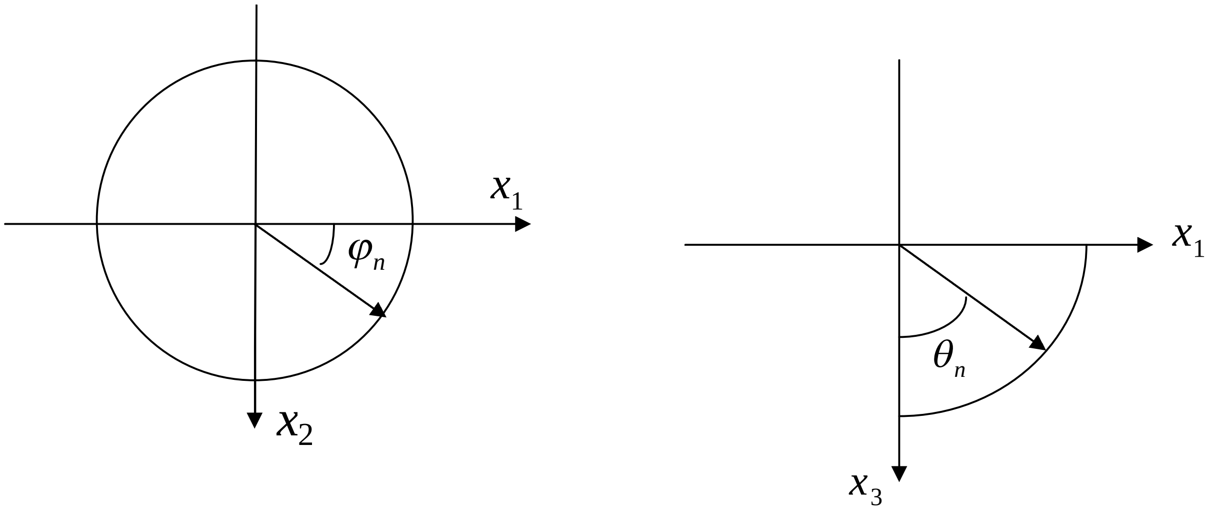
where $s(t)$ - source signal.

8.3 Source of couples forces directed along fault

8.3.1 3D Case

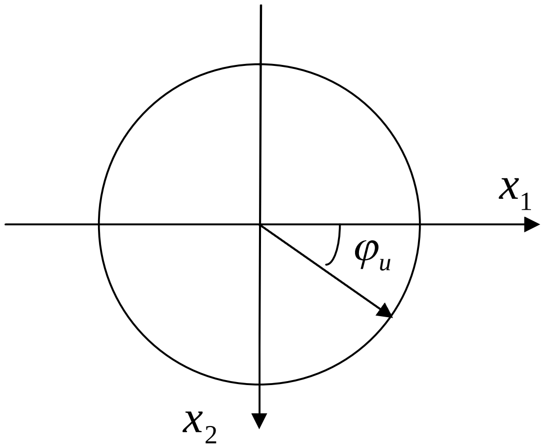
Source of coupled forces which is directed along the fault is determined by the normal $\mathbf{n}$ to the fault and jump of displacements $\mathbf{u}$ directed along the fault. Vectors $\mathbf{u}$ and $\mathbf{n}$ are always mutually perpendicular, i.e. $\mathbf{u} \cdot \mathbf{n} = 0$.

The vector $\mathbf{n}$ defined by the angle to the vertical $0 \le \theta_n \le 90^0$ and azimuth $0 \le \varphi_n < 360^0$.



The displacement vector $\mathbf{u}$ is determined differently depending on value of θ_n.

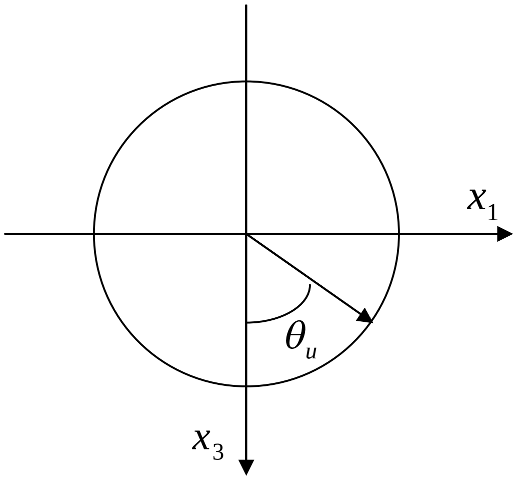
If $\theta_n \neq 90^0$, i.e. if the fault is not vertical, then vector $\mathbf{u}$ is determined by azimuth $0 \leq \varphi_u < 360^0$.



In this case, the tilt angle of the vector $\mathbf{u}$ is determined by formulae:

$$tg\theta_u = \frac{-\operatorname{ctg}\theta_n}{\cos(\varphi_u - \varphi_n)}.$$

If $\theta_n = 90^0$, i.e. the fault is vertical, then vector $\mathbf{u}$ is determined by the tilt angle $0 \leq \theta_u < 360^0$.



At this, $\mathbf{u} = (\sin\theta_u \cos\varphi_u, \sin\theta_u \sin\varphi_u, \cos\theta_u)$, где $\varphi_u = \varphi_n + 90^0$.

Directionality characteristics for the couple of forces are determined by formulae

$$L_p = \frac{a_P}{4\pi\rho V_p^3},\ L_{SV} = \frac{a_{SV}}{4\pi\rho V_S^3},\ L_{SH} = \frac{a_{SH}}{4\pi\rho V_S^3},$$

where

$$a_P = -2(\mathbf{r}\cdot\mathbf{u})\cdot(\mathbf{r}\cdot\mathbf{n})$$
$$a_{SV} = (\mathbf{r}\cdot\mathbf{u})\cdot(\boldsymbol{\theta}\cdot\mathbf{n}) + (\mathbf{r}\cdot\mathbf{n})\cdot(\boldsymbol{\theta}\cdot\mathbf{u}),$$
$$a_{SH} = (\mathbf{r}\cdot\mathbf{u})\cdot(\boldsymbol{\varphi}\cdot\mathbf{n}) + (\mathbf{r}\cdot\mathbf{n})\cdot(\boldsymbol{\varphi}\cdot\mathbf{u}),$$

$\mathbf{r} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ - polarization vector of P wave (direction to receiver),

$\boldsymbol{\theta} = (\cos\theta\cos\varphi, \cos\theta\sin\varphi, -\sin\theta)$ - polarization vector of SV wave,

$\boldsymbol{\varphi} = (-\sin\phi, \cos\phi, 0)$ - polarization vector of SH wave.

Generation of oscillations is realized by adding increments to the components of the stress tensor τ_{ij}:

$$\Delta\tau_{ij} = S(t)\cdot c_{ijpq} n_p u_q .$$

Here c_{ijpq} - components of elasticity tensor in place of the source of oscillations.

In matrix notation, which conventionally designate one pair of indices by one index $(1{,}1)\leftrightarrow 1$, $(2{,}2)\leftrightarrow 2$, $(3{,}3)\leftrightarrow 3$, $(2{,}3)\leftrightarrow 4$, $(1{,}3)\leftrightarrow 5$, $(1{,}2)\leftrightarrow 6$, for isotropic medium this equation can be denoted as

$$\Delta\tau_{ij} = s(t)\begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}\begin{pmatrix} u_1 n_1 \\ u_2 n_2 \\ u_3 n_3 \\ u_2 n_3 + u_3 n_2 \\ u_1 n_3 + u_3 n_1 \\ u_1 n_2 + u_2 n_1 \end{pmatrix},$$

where

$c_{11} = c_{22} = c_{33} = \lambda + 2\mu$, $c_{12} = c_{13} = c_{23} = \lambda$, $c_{44} = c_{55} = c_{66} = \mu$, λ, μ - Lame parameters,

$u_1 n_1 = \sin\theta_u \cos\varphi_u \sin\theta_n \cos\varphi_n$,

$u_2 n_2 = \sin\theta_u \sin\varphi_u \sin\theta_n \sin\varphi_n$,

$u_3 n_3 = \cos\theta_u \cos\theta_n$,

$u_2 n_3 + u_3 n_2 = \sin\theta_u \sin\varphi_u \cos\theta_n + \sin\theta_n \sin\varphi_n \cos\theta_u$,

$$u_1 n_3 + u_3 n_1 = \sin\theta_u \cos\varphi_u \cos\theta_n + \sin\theta_n \cos\varphi_n \cos\theta_u$$
$$u_1 n_2 + u_2 n_1 = \sin\theta_u \cos\varphi_u \sin\theta_n \sin\varphi_n + \sin\theta_n \cos\varphi_n \sin\theta_u \sin\varphi_u$$

8.3.2 2D Case

Double couple is determined by the tilt angle of the normal θ_n to the fault $\mathbf{n} = (\sin\theta_n, 0, \cos\theta_n)$. Displacement vector $\mathbf{u}$ is directed along fault and perpendicular to $\mathbf{n}$. It has coordinates $\mathbf{u} = (\cos\theta_n, 0, -\sin\theta_n)$.

Directionality diagrams of compression and shear waves are described by formulae

$$L_p = \frac{\sin 2(\theta - \theta_n)}{4\pi\rho V_p^3}, \; L_{SV} = \frac{\cos 2(\theta - \theta_n)}{4\pi\rho V_p^3}.$$

Signal generation is done by adding increments of time to the stresses τ_{11}, τ_{13}, τ_{33} accordingly to formulae:

$$\Delta\tau_{11} = s(t)\mu \sin 2\theta_0,$$
$$\Delta\tau_{13} = s(t)\mu \cos 2\theta_0,$$
$$\Delta\tau_{33} = -s(t)\mu \sin 2\theta_0.$$

8.4 Point source with an arbitrary moment tensor M

Moment tensor $\mathbf{M} = (m_{ij})$ is described by symmetric matrix of size 3×3. Consequently it is defined by 6 numbers.

Symmetric matrix can always be diagonalized by 3 rotations. The coordinate system in which the $\mathbf{M}$ is diagonal is called the own, and the directions of the axes - the main ones. In this coordinate system the tensor is defined by 3 numbers $m_{11}^{(0)}$, $m_{22}^{(0)}$ and $m_{33}^{(0)}$. The remaining values $m_{ij}^{(0)} = 0$ (when $i \neq j$).

In its own coordinate system, the tensor effect is described by three vector dipole directed along the axes of the coordinate system and have intensity $m_{11}^{(0)}$, $m_{22}^{(0)}$, $m_{33}^{(0)}$. They are tensile or compressive stresses.

To move from their own coordinate system to the external, we will use 3 turns with Euler angles, i.e. will use the angles: α - precession angle, β - the angle of nutation and γ - the angle of own rotation.

Rotation of matrix $\mathbf{M}^{(0)} = (m_{ij}^{(0)})$ is executed by multiplying on rotation matrixes:

$$\mathbf{M} = \mathbf{T}_\alpha \mathbf{T}_\beta \mathbf{T}_\gamma \mathbf{M}^{(0)} \mathbf{T}_\gamma^* \mathbf{T}_\beta^* \mathbf{T}_\alpha^*, \tag{1}$$

where $*$ denotes transposing,

$$\mathbf{T}_\alpha = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; \mathbf{T}_\beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{pmatrix}, \; \mathbf{T}_\gamma = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

In the program angles α, β, γ are defined in relating ***dialog*** in tree steps:

1. Rotation around the OZ axis (i.e. in planes parallel to XY) at an angle α from the axis OX to the axis OY. The resulting line of nodes is stretched along OX axis.

2. Rotation of the XY plane around the line ON of nodes by angle β. The axis OZ will also be turned at angle β and take the position $OZ^{(0)}$.
3. Rotation of a new position of XY plane around the $OZ^{(0)}$ axis at an angle γ. The line ON of nodes will turn and will assume the position $OX^{(0)}$. The axis OY will change the position to $OY^{(0)}$.

Coordinate system $\left(X^{(0)}, Y^{(0)}, Z^{(0)}\right)$ is what we were looking for (own). Within this coordinate system 3 numbers $m_{11}^{(0)}$, $m_{22}^{(0)}$ и $m_{33}^{(0)}$ are determined. Затем, осуществляется поворот по формуле Then, rotation is executed accordingly to the formula (1).

Initialization of the source is done by the program by to the stress tensor τ_{ij} of increments $s(t) \cdot m_{ij}$, where $s(t)$ - is source signal.

8.5 Modeling of radiation, caused by shifts along faults in Tesseral 2.5D and 2D software

Currently methods of tracking of changes in reservoir properties using the recorded randomly radiated, which are excited in the area of fracture (seismic emission) become widely used.

These techniques are based on the principle of interferometry. However predicting how to plan your observing system and which processing procedure must be used is still quite complex task.

These issues can be resolved at simulating of the spontaneous emission from the fracture zone. The simulation can be done either in 2.5D medium model case for arbitrary system of vertical fractures or fissures 2D systems, or in 2D case for systems of fractures located within planes parallel to the plane $X_1 X_3$ or planes parallel to $X_2 X_3$.

In 2.5D case the fault can be presented as an arbitrary surface $F(x, y, z) = 0$, and the shift – by some 3D-vector $\mathbf{v} = (x_0, y_0, z_0)$. Method of simulating of the sources distributed on some surface consists in splitting them line - cross-sections $F(x, y_S, z) = 0$ with a constant lateral offset $y_S = const$. All sources allocated along this line can be initialized simultaneously, but with different signals. To obtain signals of reflected waves recorded by the receiver with side offset y_R, in the Fourier transformation (the same formula as that of a point source) must be used distance $y_R - y_S$.

If the surface point $\mathbf{x} = (x_S, y_S, z_S)$ belongs to the surface $F(x, y, z) = 0$ and is the source, it is necessary in this point to find a normal, which is the gradient of function $F(x, y, z)$: $\mathbf{n} = gradF = (\partial F / \partial x, \partial F / \partial y, \partial F / \partial z)$ and the plane perpendicular thereto (the tangent to the surface) and project vector $\mathbf{v} = (x_0, y_0, z_0)$. To do this, vector $\mathbf{p} = \mathbf{v} - (\mathbf{v} \cdot \mathbf{n}) \cdot \mathbf{n} / |\mathbf{n}|^2$ and then normalized as $\mathbf{m} = \mathbf{p} / |\mathbf{p}|$. We assume that along the tangent plane originated jump of displacements $[u_i] = \mathbf{m}$, leading to the source of the seismic moment (see Aki, Richards, Quantitative Seismology, page 57).

The position of vertical plane fractures can be specified by the azimuth φ of their normals $\mathbf{n} = (\cos\phi; \sin\phi; 0)$. Along the fracture planes can occur of jumps of the fractures displacement

$[u_i] = u_i^+ - u_i^-$, leading to sources of a seismic moment M_{ij} type (see Aki, Richards, Quantitative Seismology, page 57). Displacement jumps within the fault plane are described by vectors $[u] = a(-\sin\varphi\sin\alpha; \cos\varphi\sin\alpha; \cos\alpha)$ perpendicular to the normal $\mathbf{n}$ where α is the angle from the vertical, with the amplitude a of the jump.

Denote λ_{ijpq} as the elasticity tensor in the region of the fracture, break it down into square cells, and apply modeling the radiation sources at the nodes of the grid.

According to this model, in each grin node there is a source of seismic moment $M_{pq} = [u_i] n_j \lambda_{ijpq}$.

For a fixed receiver $P(x_{1P}, x_{2P}, x_{3P})$ the sources arranged along a vertical line $x_{1S} = const$, $x_{2S} = const$ have the same side offsets $x_2 = x_{2P} - x_{2S}$ relatively to the receiver P. Consequently, for the 2.5D scheme sources on the vertical line can be taken into account if they generate signals synchronously and at the step of the inverse Fourier transform $\sum_{k_2} f(\omega) e^{j\omega k_2 x_2}$ to recover the correct signal at the reception point. Since the vertical lines of sources are located along a fixed azimuth $\varphi + 90^0$ and given the invariant properties of 2.5D medium in the direction x_2, it is possible to project all the sources on the OX_1 axis, and then properly take into account the side offsets from the receiver at the synthesis of the final sum of the signal from the source (Fig. 1).

Thus, the synthesis of the signal caused by braking along the plane tangent to the fracture is reduced to simulation of a set of sources of line type (as in the 2D simulation of emitting boundary line) with the coordinates $x_1, ..., x_n$ and the subsequent accumulation for each receiver of incoming events.

Synthesis of signals for each source on the line is executed by adding the increment of the signals stresses $f(t)$ multiplied by the seismic moments M_{ij}:

$$\Delta\tau_{ij} = M_{ij} f(t),$$

where $f(t)$ is the source signal.

2D modeling of emissions caused by shifts along faults, is a special case of discussed above 2.5D-modeling. In this case it is necessary to assume that the fracture has a predetermined section within plane X_1X_3, and does not change in the OY axis direction. It is defined by a line-section defined by the polygon. We need to find the normal and tangent to the line of the polygon at the source and use the formula given above. Additional displacement vector determining is not necessary

If the fracture is within the plane X_1X_3 (or rather the task is posed for the system of fractures with a constant density in direction x_2), it is necessary to perform the synthesis of signals from uniformly distributed sources of seismic moments in the plane (Fig. 2). Here the displacement vector is not needed.

Point seismic moments M_{ij} are calculated in the same way as for 2.5D case, but for the vertical plane X_1X_3.

Moments M_{ij} can be expressed as a linear combination of the nine possible pairs of forces L_{pq} represented by Figure 3.7 in the Aki Richards book. Implementation point source producing L_{pq} moment is reduced to form the excitation signal at the point using formulae $\Delta\tau_{pq} = L_{pq}$.

This task is much easier than task of forming the signals from the surface of the fracture. However, such seismic sources can be used only for the wave field in the far zone of fracture of a small size compared to the distance to the receiver.

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